Deterministic and Stochastic Brain Rhythms

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The noise may *directly* affect:

– a variable of the system we are observing
– a parameter of the system
– the derivatives of the variables of the system
– all of the above

Noise can act

- in time: temporal noise
- in space: spatial noise
- both: spatiotemporal noise.

BRAIN RHYTHMS

ONGOING VOLTAGE FLUCTUATIONS in neuron populations

*Local Field Potentials (LFP)
*Electro-Encephalogram (EEG) of Magneto-Encephalogram (MEG)
*MRI + EEG +MEG for very slow rhythms

Gamma (30-100 Hz): cognitive phenomena (memory, attention…)

Beta (10-30 Hz): eyes open; thinking, suppressed before movement changes

Alpha (8-12 Hz): at rest with eyes closed, REM

Theta (6-10 Hz): active/exploratory motor behavior, memory formation, REM

Delta (0.5-4 Hz): deep sleep stage 3 (slow wave sleep)

Infraslow (< 0.1 Hz): epochs of conscious percepts or thoughts (?)

Brain Rhythm Characteristics

- Frequency range
- Amplitude (delta is the strongest)
- Frequency content vs time (spectrogram)
- Variance of frequency and amplitude
- Phase spectrum
- Power law exponent(s) in the Power Spectrum? Other form?
- Entropy, recurrence, fractal dimensions, Lyapunov spectrum, critical exponents, …
- Burstiness: amplitude and durations of epochs of oscillation

Brain Rhythm "STATISTICS" are altered in Brain disorders:

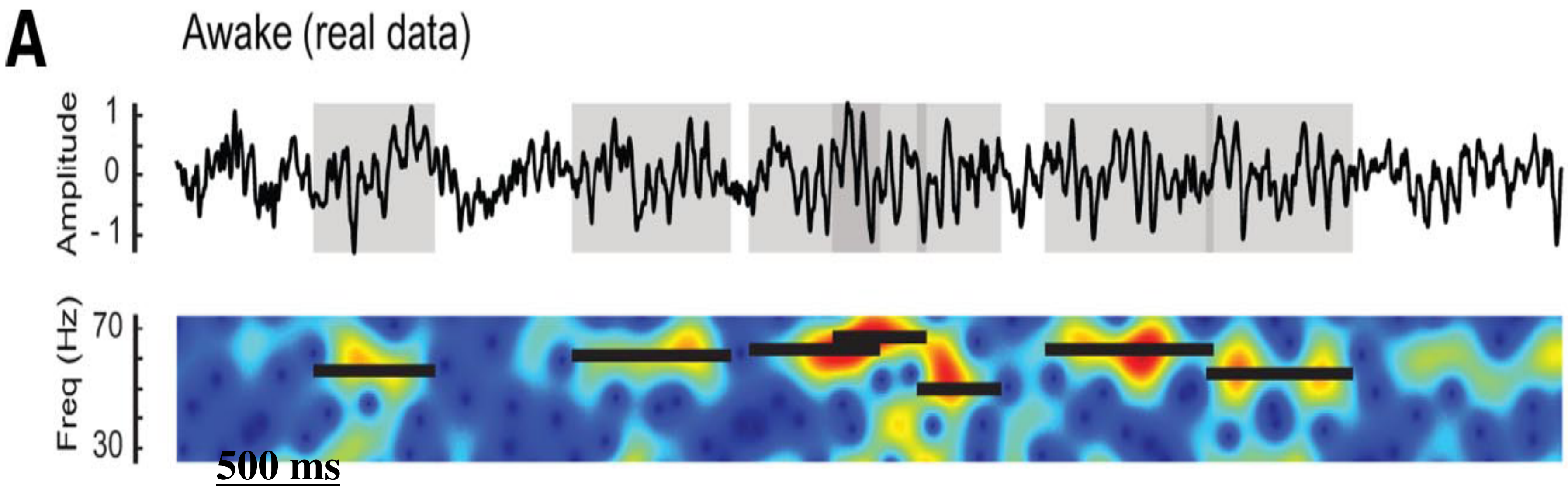
Parkinson's, Epilepsy, Schizophrenia, Autism …

Example of gamma rhythm:

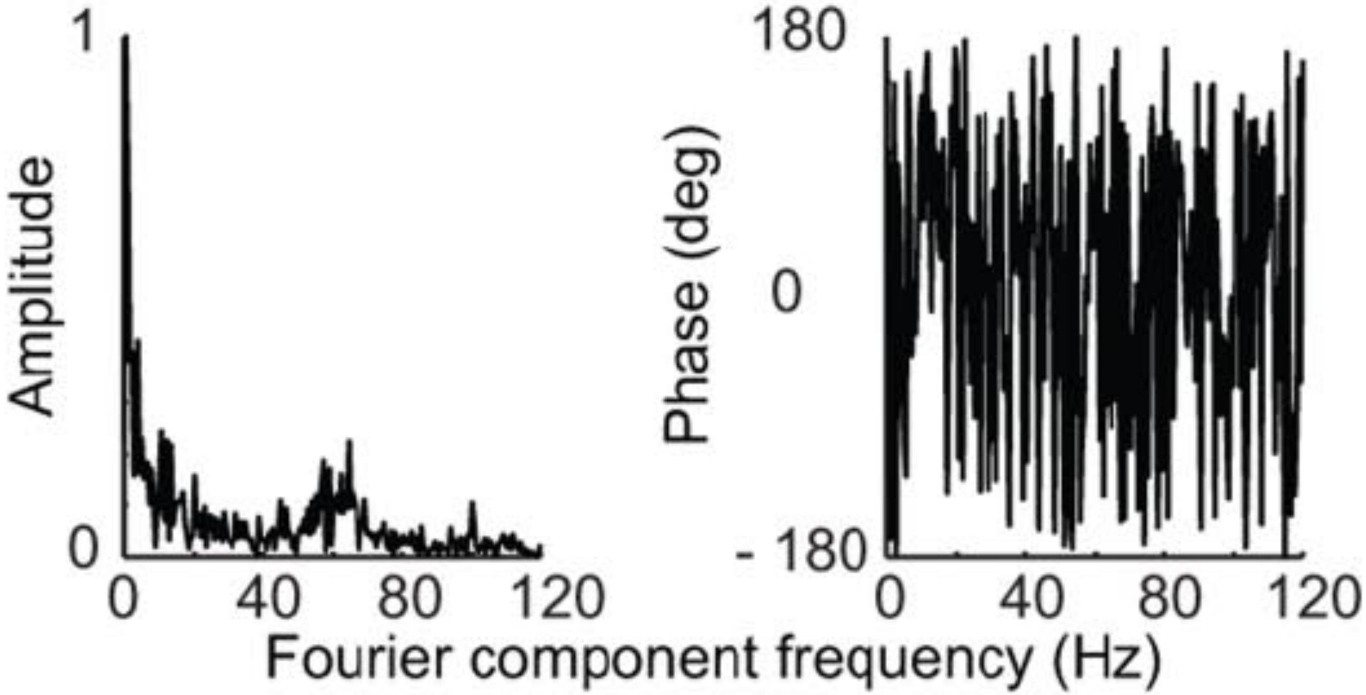
Data from Visual Cortex of macaque (Xing et al, J.Neurosci. 2012)

TOP: VOLTAGE FROM RECORDING ELECTRODE

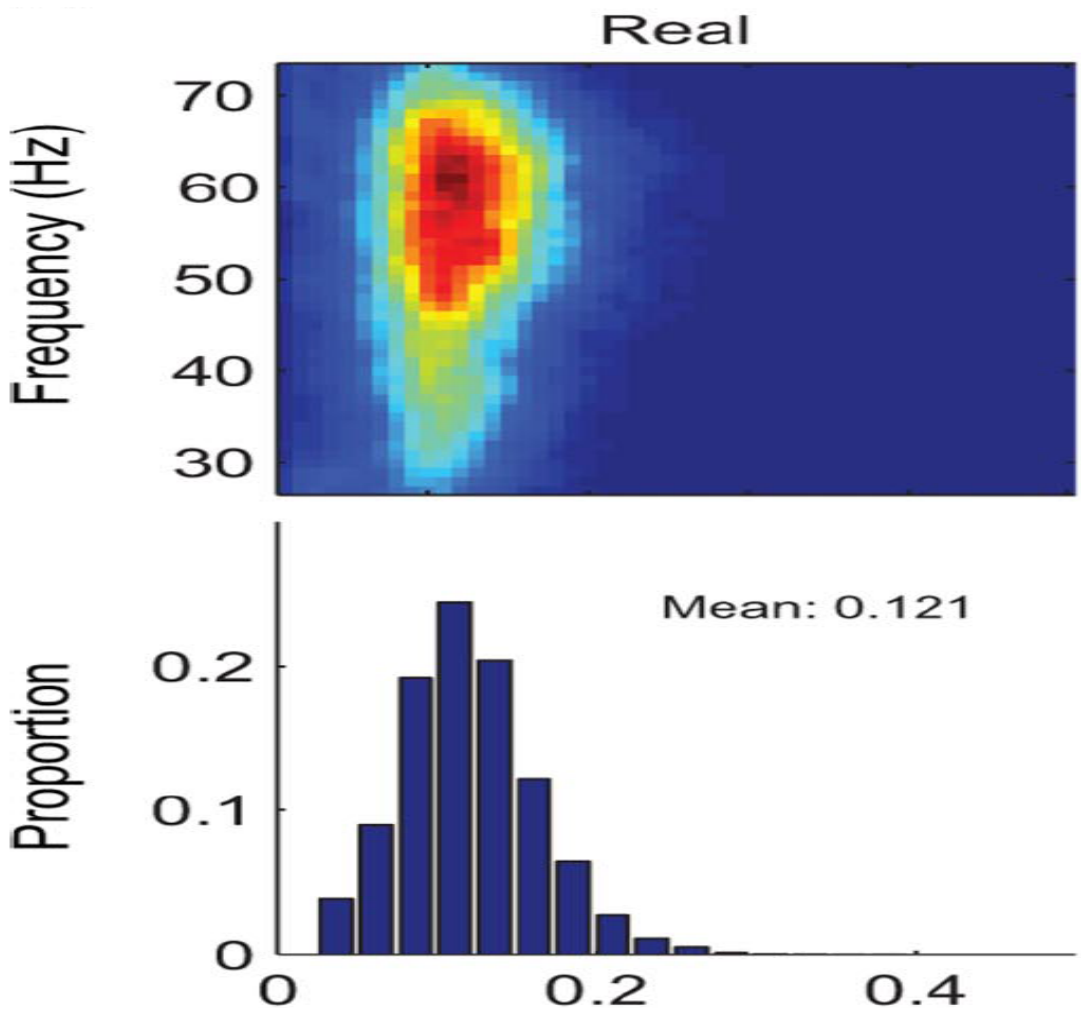
BOTTOM: TIME-FREQUENCY REPRESENTATION (spectrogram)



FREQUENCY CONTENT OF STOCHASTIC RHYTHM
(via Fourier Analysis)



PEAK FREQUENCY CONTENT DURING BURST
vs
BURST DURATION
(Xing et al. 2012)



BURST DURATION (SEC)

Beta bursts in Local Field Potentials from the motor cortex in a single trial

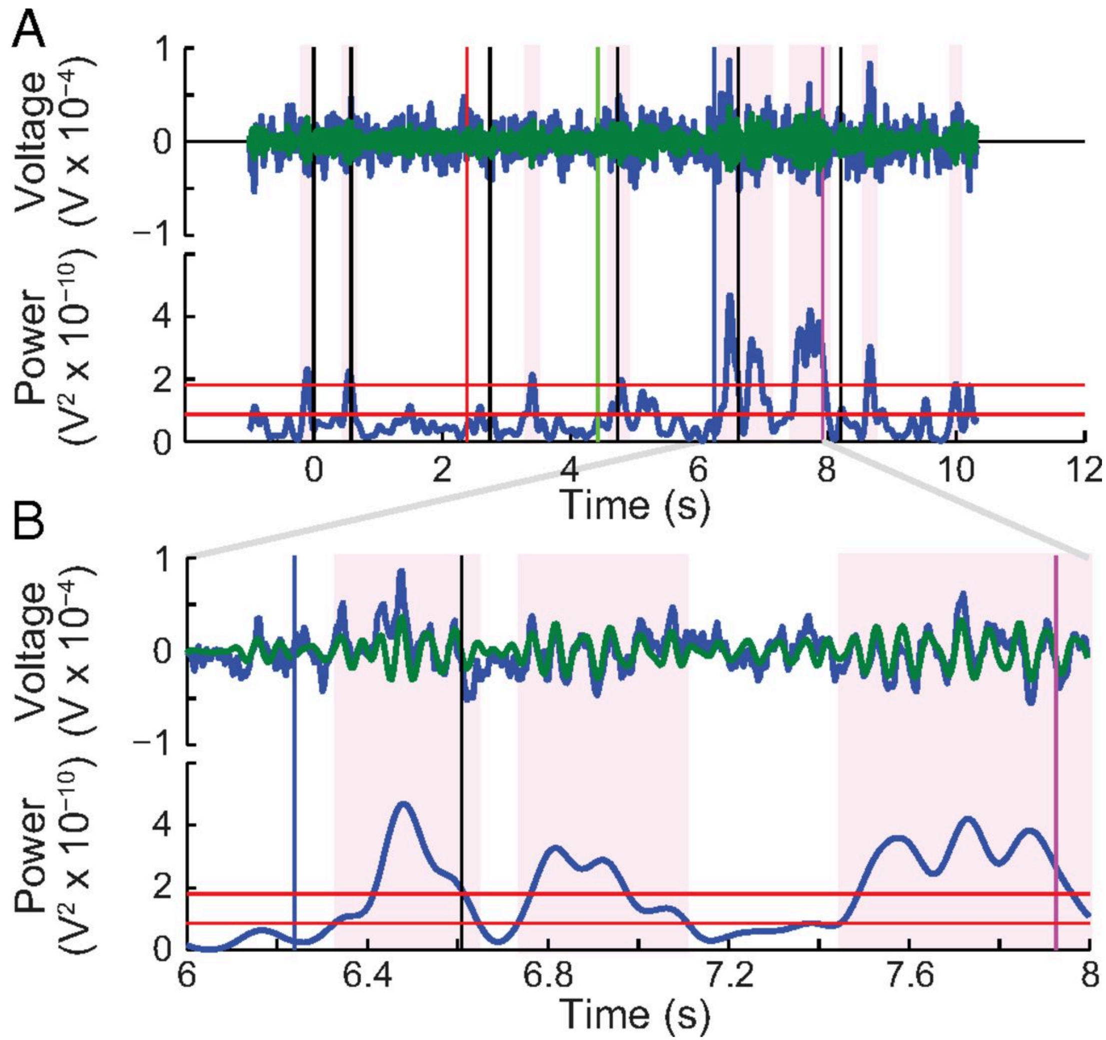
Blue: raw LFP
Green: beta bandpass-filtered LFP

Dark Blue: beta power

<u>ZOOM IN</u>

Lower threshold:
1.5 times median power

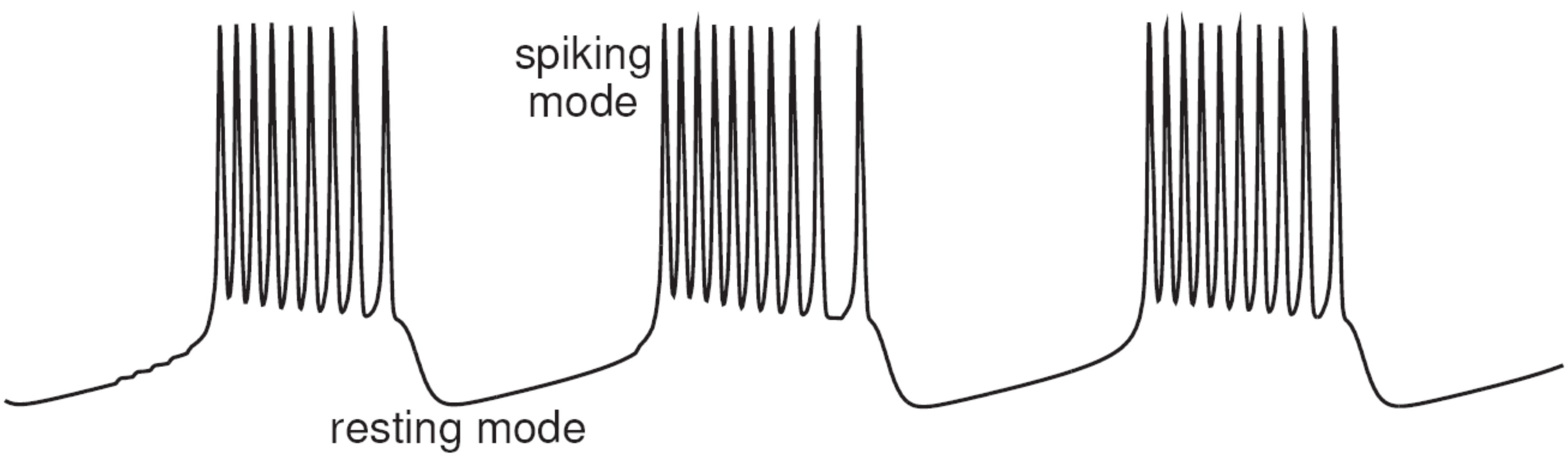
→ used to define bursts



Joseph Feingold et al. PNAS 2015

- So rhythms do not look like clean sine waves
- They have complex shapes, and appear strongly “contaminated” by noise: intrinsic? recording artefact?
- In fact they appear to be non-stationary – probably a good thing!

Some cells are endogenous bursters:
do they **cause** brain rhythms? → they can



Izhikevich.com

Supercritical Andronov-Hopf

frequency non-zero at onset, does not vary much

Izhikevich.com

BRAIN RHYTHMS: Network Model for their generation

- Two principal types of neuron: Excitatory (E) and Inhibitory (I)
- Oscillation requiring E and I: PING
- Oscillation requiring I only (and E follows): ING
- Rhythms autonomous, or induced by, or altered by stimulation
- BUT: REAL RHYTHMS ARE STOCHASTIC (maybe also chaotic)

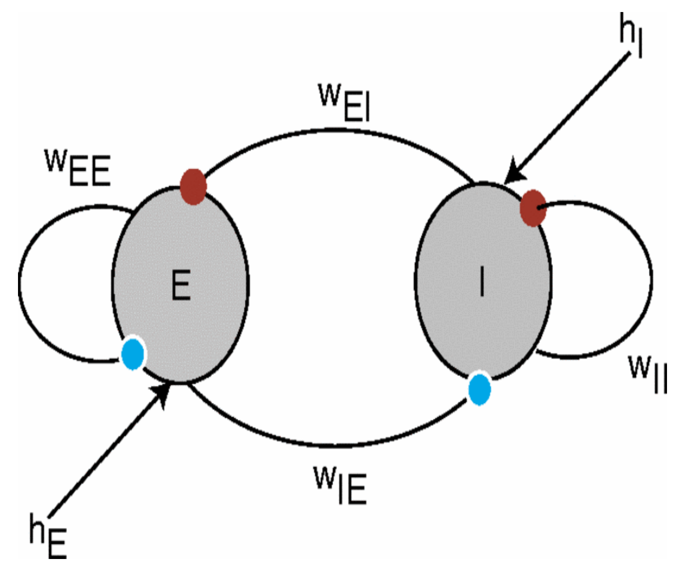
Stochastic Wilson-Cowan model for oscillation generation

(Wallace et al. 2015)

$$\frac{dE(t)}{dt} = -\alpha_E E(t) + \big(1 - \mathrm{E(t)}\big)\beta_E f(S_E) + \sqrt{\frac{\alpha_E E(t) + \big(1 - \mathrm{E(t)}\big)\beta_E f(S_E)}{N_E}}\, \xi_E(\mathrm{t})$$

$$\frac{dI(t)}{dt} = -\alpha_I I(t) + \big(1 - I(\mathrm{t})\big)\beta_I f(S_I) + \sqrt{\frac{\alpha_I I(t) + \big(1 - I(\mathrm{t})\big)\beta_I f(S_I)}{N_I}}\, \xi_I(\mathrm{t})$$

$$S_E(t) = W_{EE}E(t) - W_{EI}I(t) + h_E$$
$$S_I(t) = W_{IE}E(t) - W_{II}I(t) + h_I$$



$\xi_E(t)$, $\xi_I(t)$: two independent Gaussian white noises

Complex conjugate eigenvalues

$\lambda = -\nu \pm j\,\omega_o$

Stochastic Wilson-Cowan model for oscillation generation

(Wallace et al. 2015)

Wilson-Cowan | Finite size noise

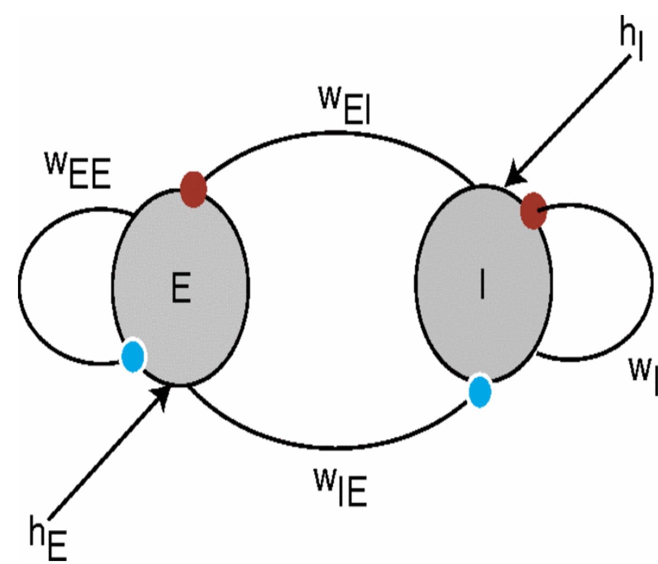
$$\frac{dE(t)}{dt} = -\alpha_E E(t) + \left(1 - E(t)\right)\beta_E f(S_E) + \sqrt{\frac{\alpha_E E(t) + \left(1 - E(t)\right)\beta_E f(S_E)}{N_E}}\,\xi_E(t)$$

$$\frac{dI(t)}{dt} = -\alpha_I I(t) + \left(1 - I(t)\right)\beta_I f(S_I) + \sqrt{\frac{\alpha_I I(t) + \left(1 - I(t)\right)\beta_I f(S_I)}{N_I}}\,\xi_I(t)$$

Synaptic input

$$S_E(t) = W_{EE}E(t) - W_{EI}I(t) + h_E$$
$$S_I(t) = W_{IE}E(t) - W_{II}I(t) + h_I$$

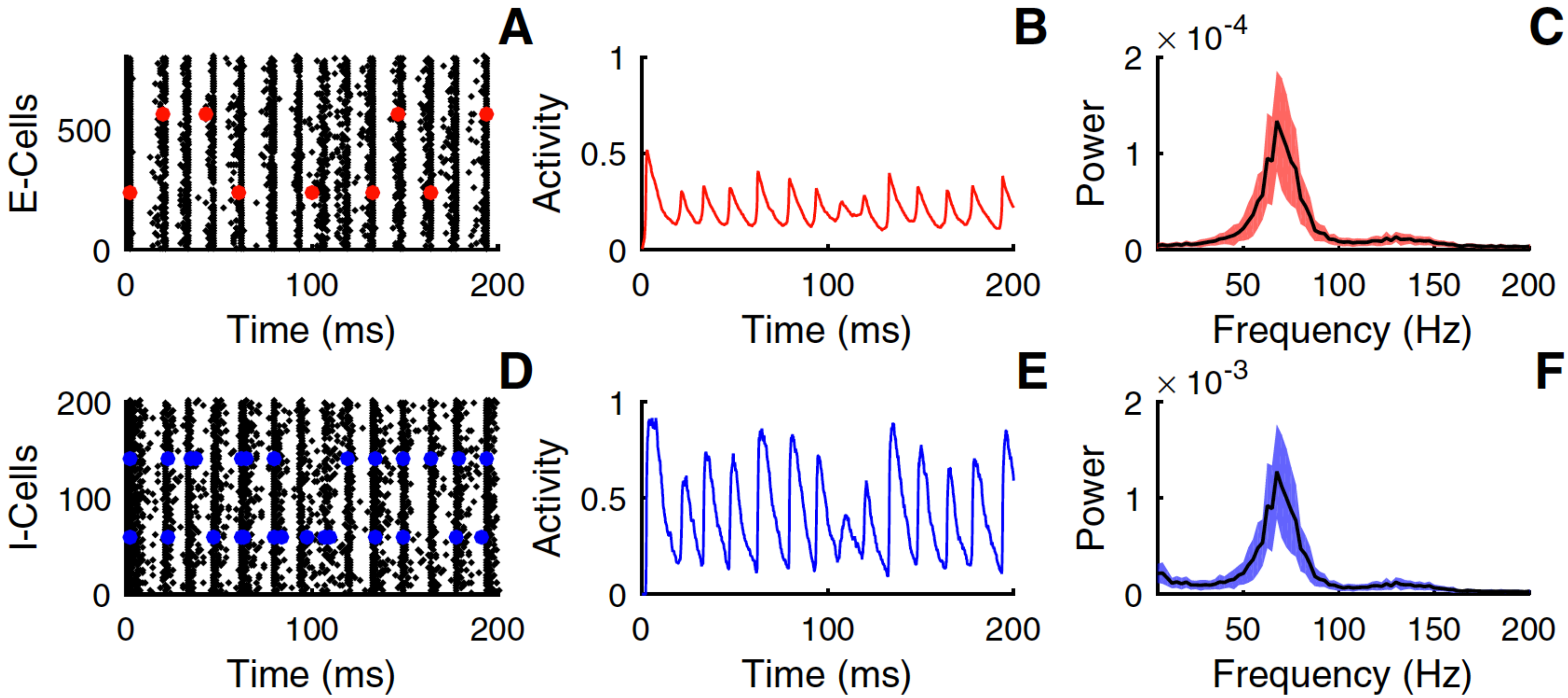
$\xi_E(t)$, $\xi_I(t)$: two independent Gaussian white noises



Complex conjugate eigenvalues

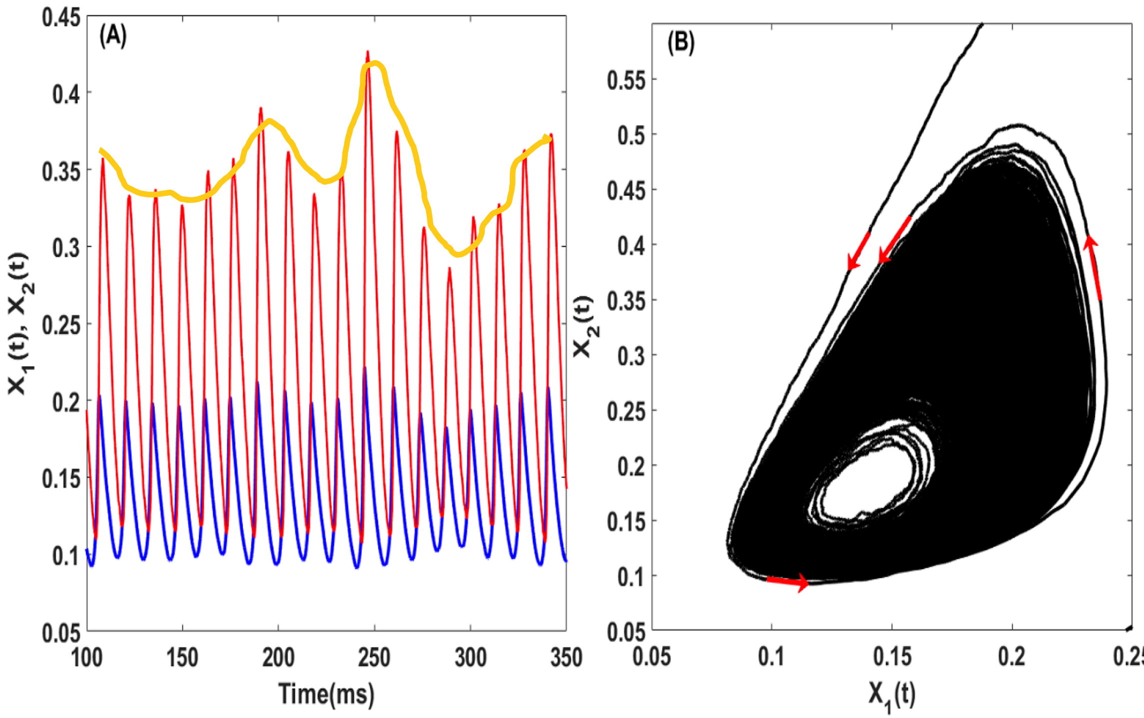
$\lambda = -\nu \pm j\,\omega_o$

E-I rhythm:
the cells do not fire at every cycle (asynchronous regime)



Dumont, Northoff, Longtin, JCNS2015

Noisy Brain Rhythms



Arthur Powanwe

- Gamma oscillations *in vivo* appear as random short epochs

  → "gamma bursts"

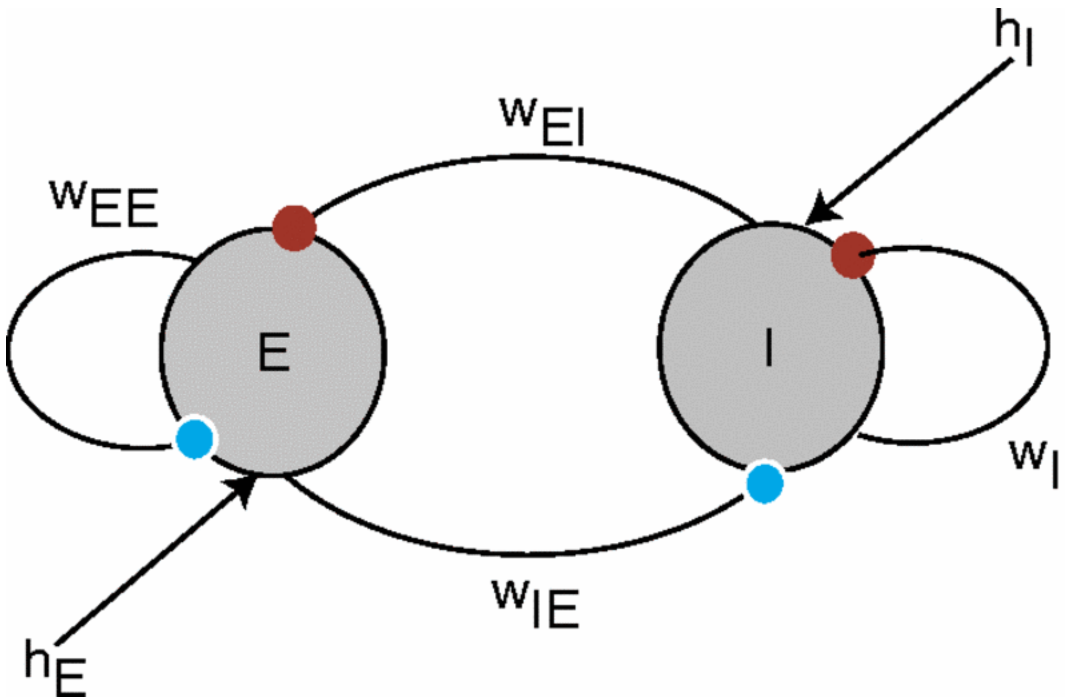
- Mean burst duration: 65-150 ms

- Gamma activity *in vivo*:

→ Broadband noise filtered by E-I network

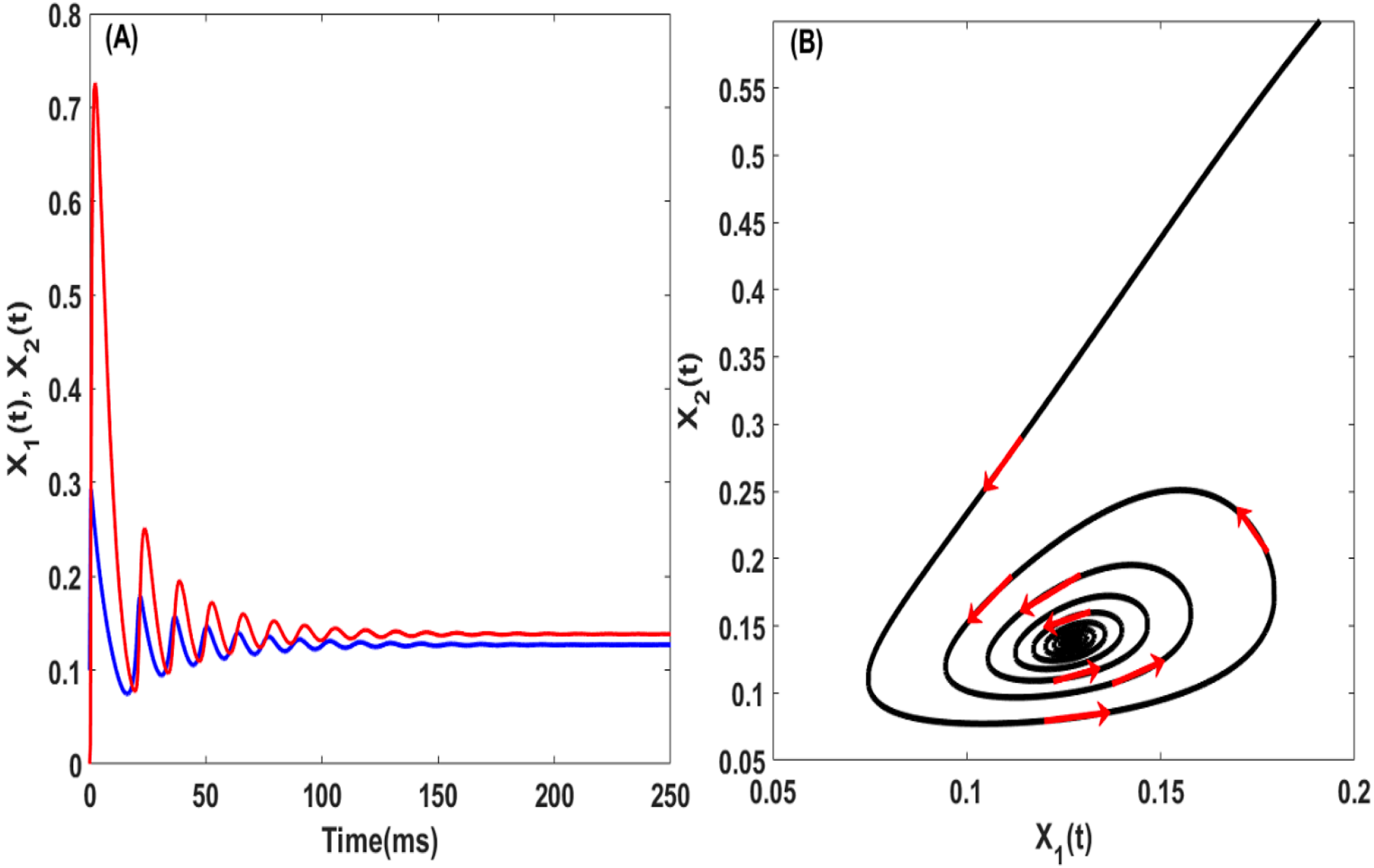
- Bursts may be important communication events (Palmigiano et al, 2017)

- Bursts altered in pathology

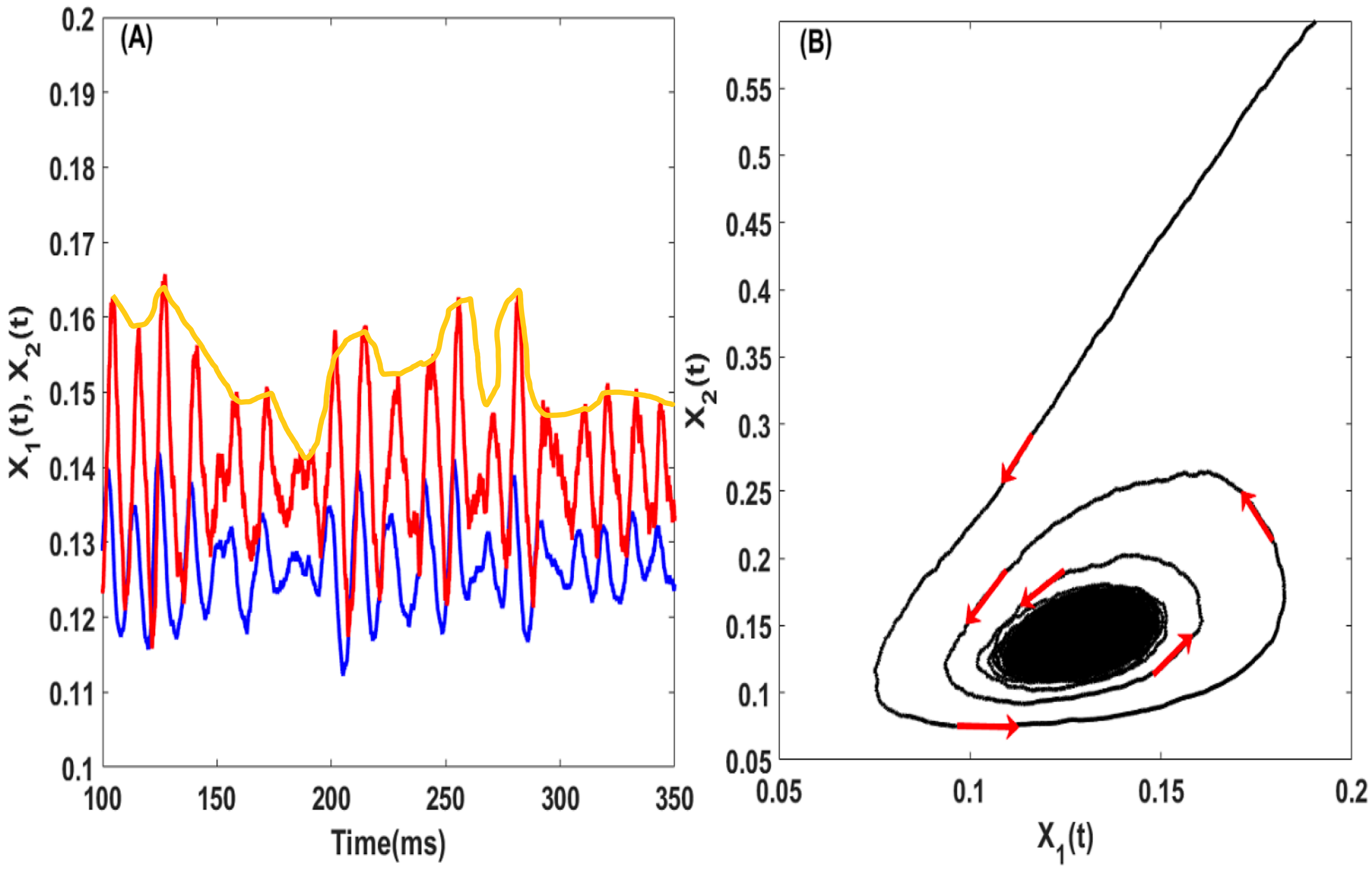


h_I
w_{EI}
w_{EE}
E
I
w_{II}
w_{IE}
h_E

Fixed point with complex eigenvalues

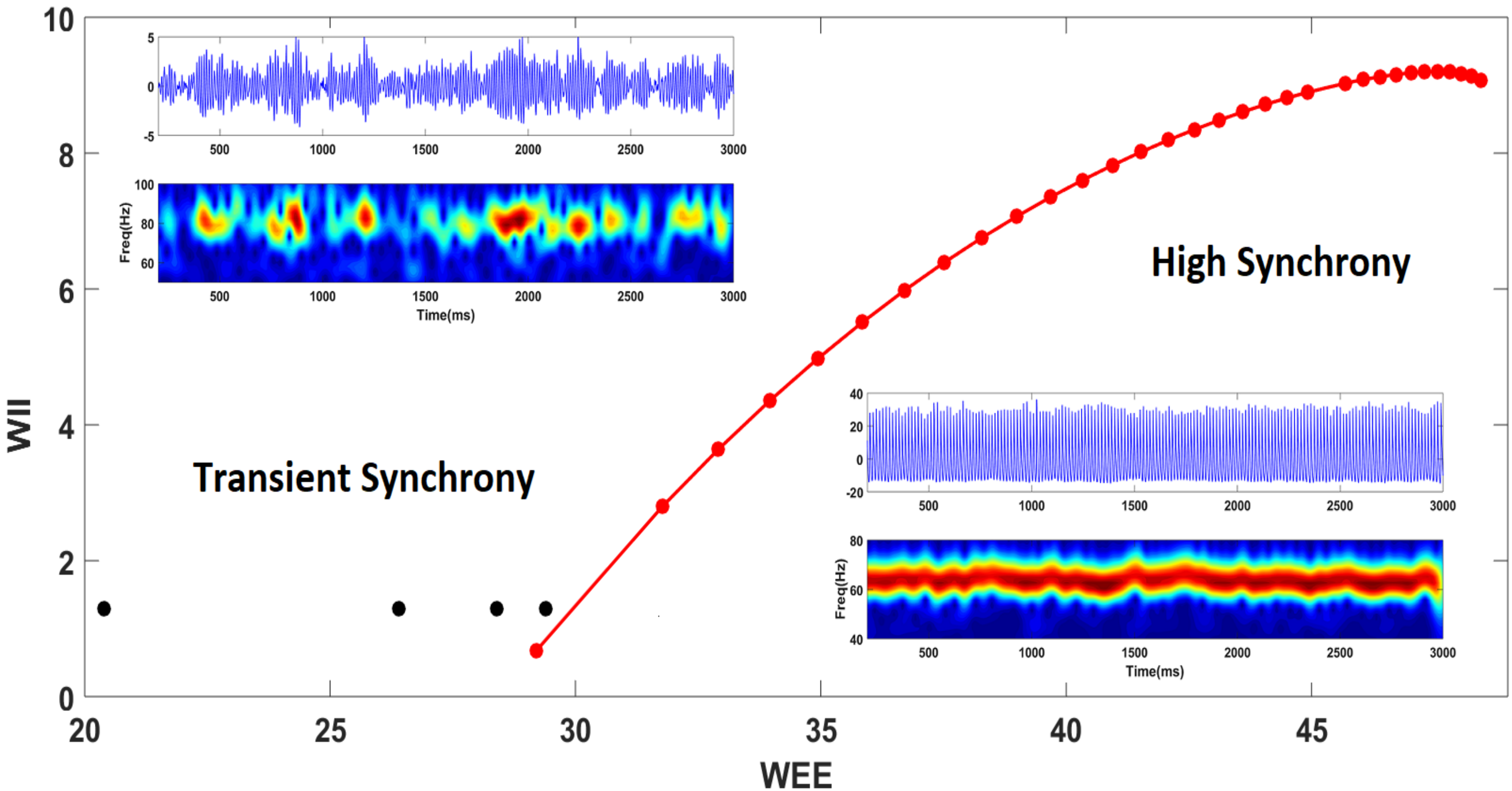


Fixed Point with Noise: Quasi-Cycle



Transient and High Synchrony Regimes

(Powanwe–Longtin, Sci Rep. 2019)



Optimal range seems to be quasi-cycle regime
(see SanCristobal et al. Nature Phys. 2015)

Transient synchrony regime

→ near but below onset of synchrony

→ no deterministic limit cycle

→ noise-induced rhythm

Envelope-phase equations via "Stochastic Averaging Method"

Roberts, J. & Spanos, P. Stochastic averaging: an approximate method of solving random vibration problems. Int. J. Non-Linear Mech. (1986)

$$dZ_E(t) = \left(-\nu Z_E(t) + \frac{D}{2Z_E(t)}\right)dt + \sqrt{D}\, dW_1(t)$$

$$d\phi_E(t) = \frac{\sqrt{D}}{Z_E(t)}\, dW_2(t)$$

$$\nu = -\frac{A_{11} + A_{22}}{2},\ \omega_0 = \frac{1}{2}\sqrt{-(A_{11} - A_{22})^2 - 4A_{12}A_{21}} \quad \text{and} \quad D = -\frac{A_{12}}{2\omega_0^2}\left(-A_{12}\sigma_I^2 + A_{21}\sigma_E^2\right)$$

Powanwe and Longtin, Scientific Reports 2019

See also Greenwood-McDonnell-Ward, NECO 2015

Envelope Probability $P(Z_E)$
→ Z_E is high during a burst

$$P(Z_E) = \left(\frac{2\nu}{D}\right) Z_E \exp\left(-\frac{\nu}{D} Z_E^2\right)$$

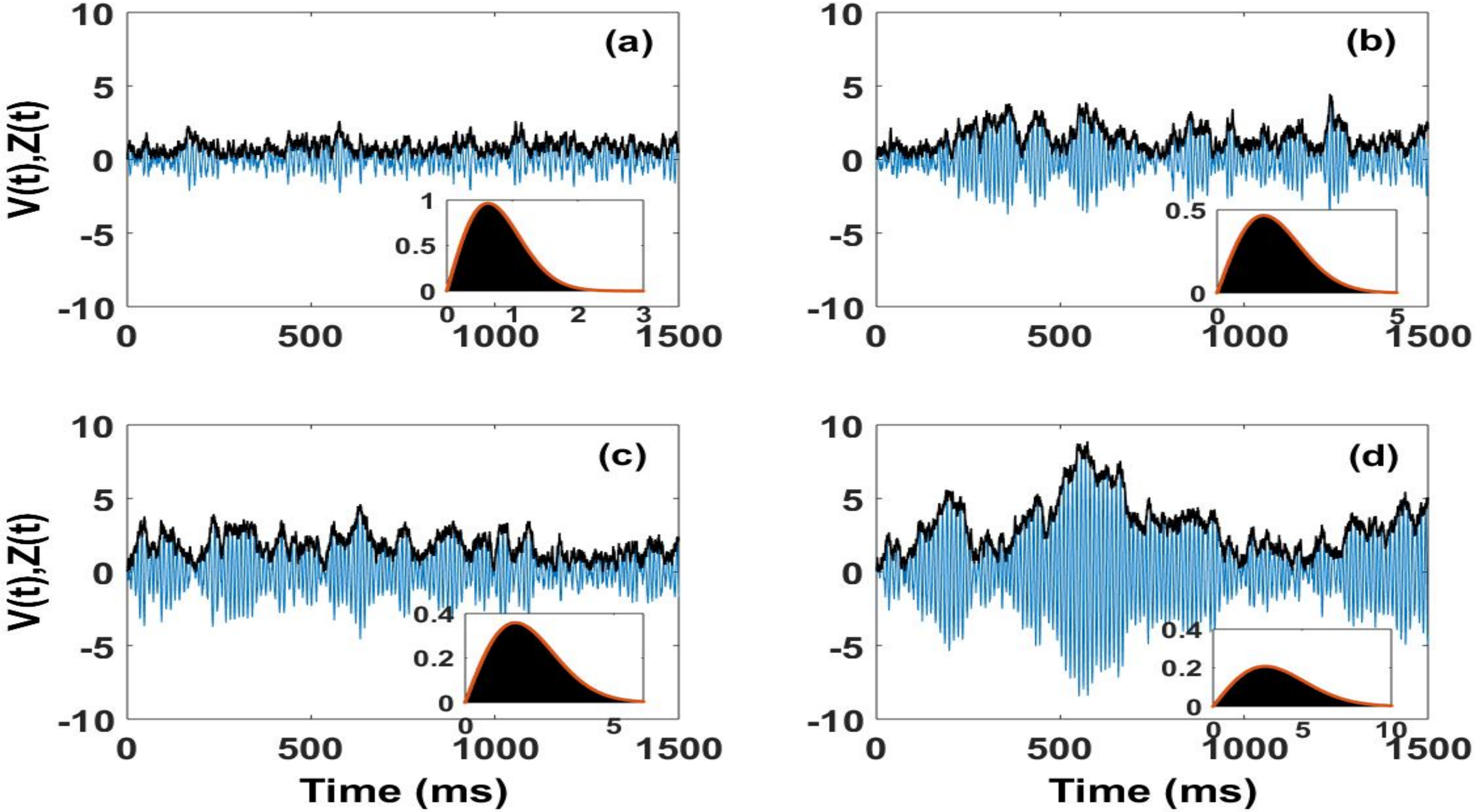
which has its peak at

$$R \equiv \sqrt{\frac{D}{2\nu}}$$

Powanwe and Longtin, Scientific Rep. 2019

You can compare this result to the statistics of the envelope extracted using the Hilbert Transform (see computer codes in tutorial)

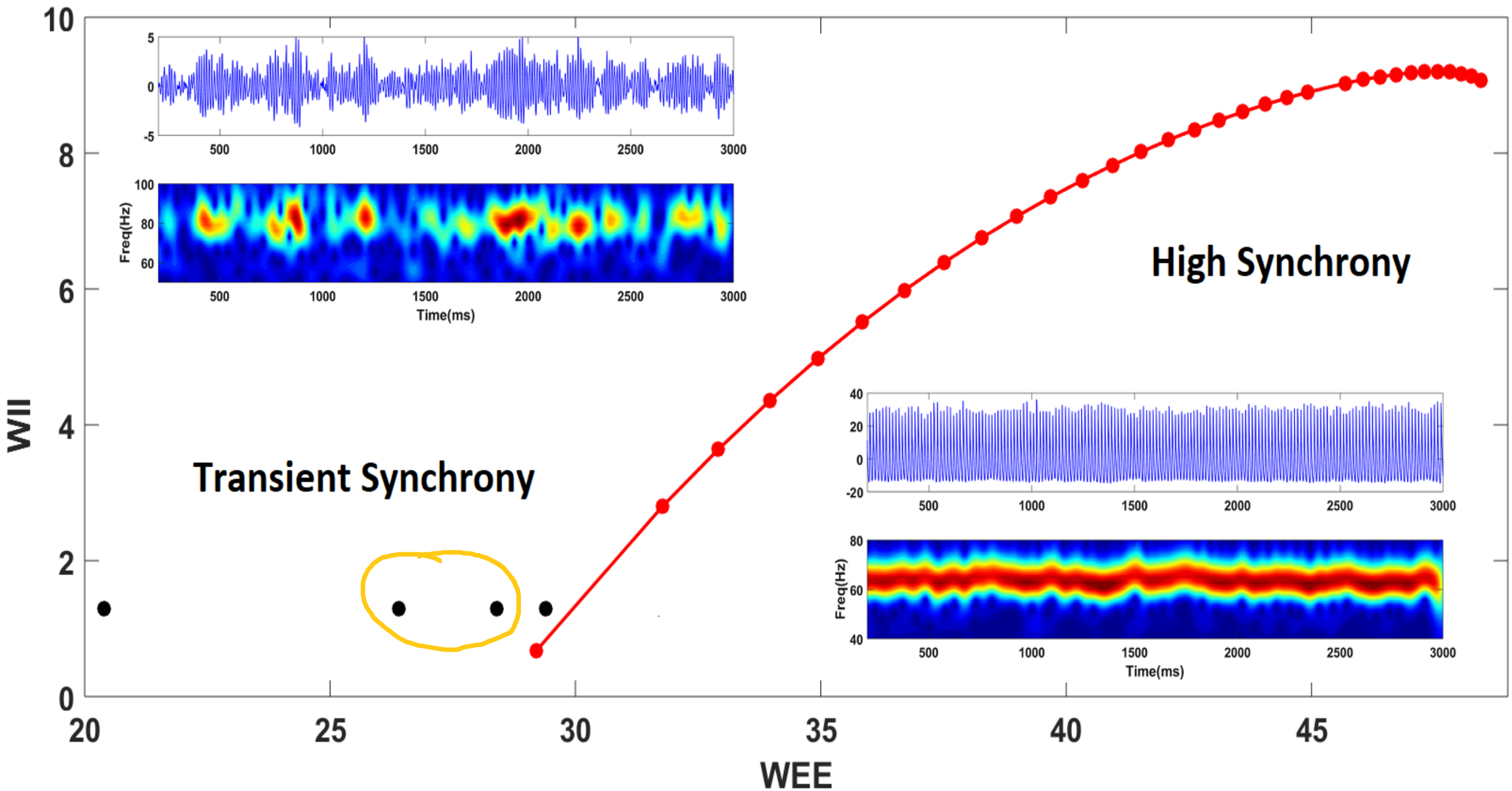
LFP dynamics in the parameter space



Theory of Rhythm Bursts

- Noise-induced rhythms
- Burst duration increases approaching Hopf bifurcation
- Duration governed by one meta-parameter
- Correct a pathology by bringing the parameters in the right range

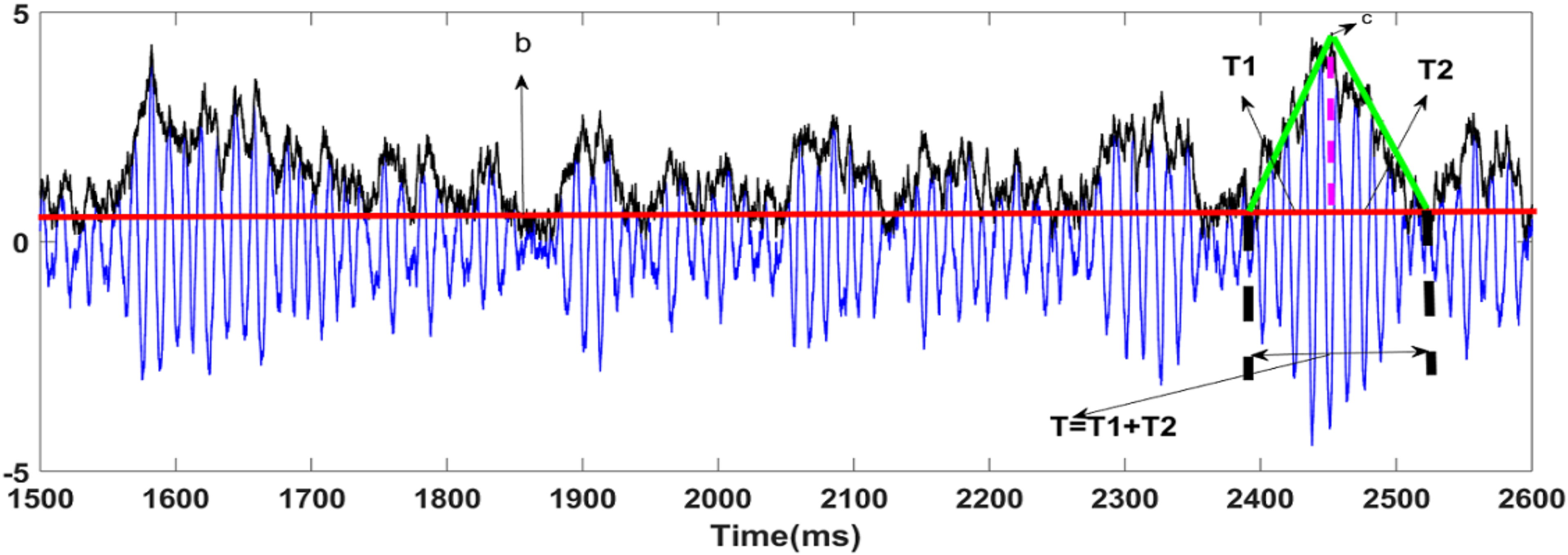
Transient and High Synchrony Regimes



Optimal range seems to be quasi-cycle regime
(see SanCristobal et al. Nature Phys. 2015)

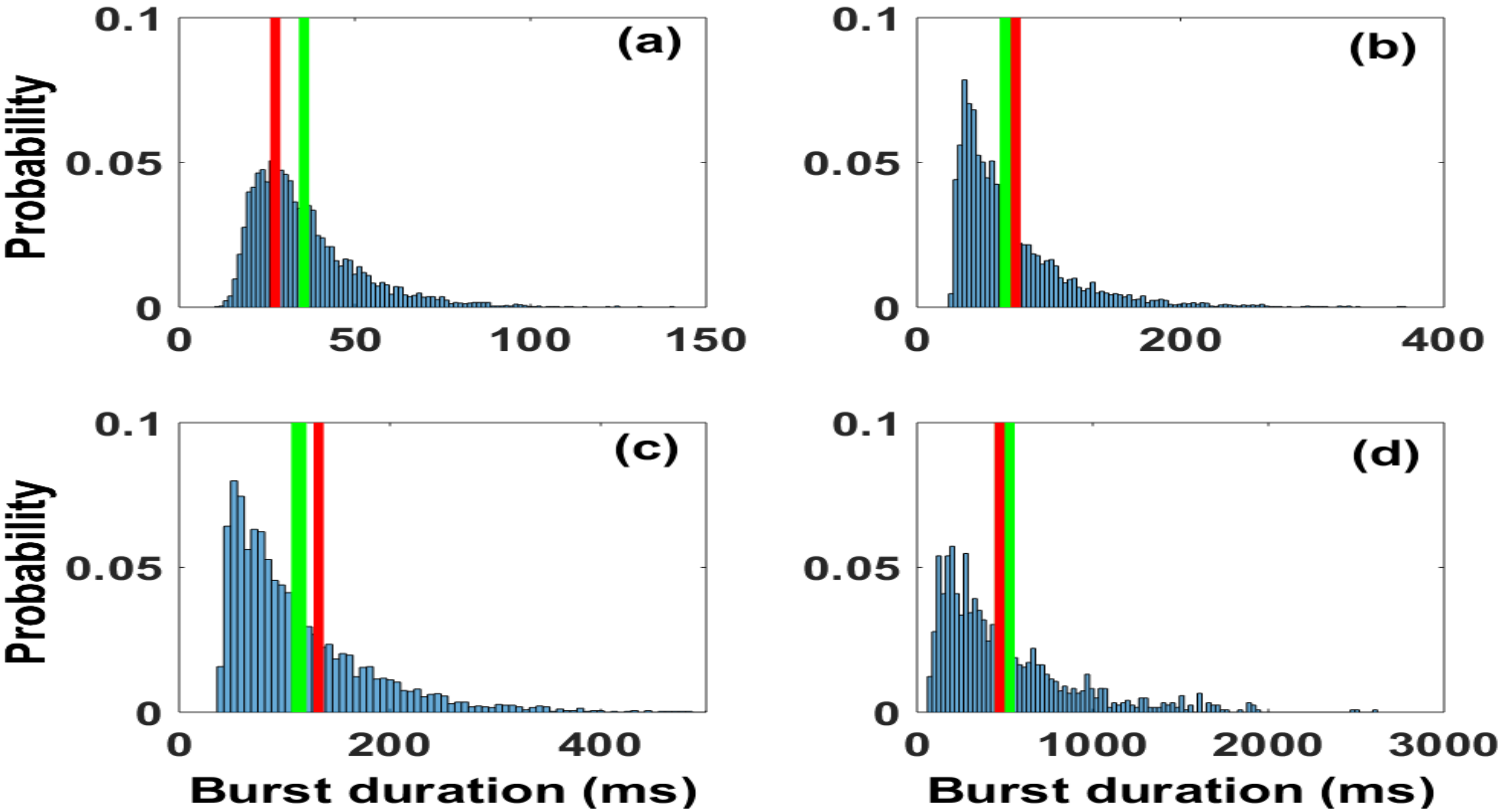
Mean Burst duration from the Fokker Planck formalism

- $T=\left(\frac{1}{2\mu}\right)\left[exp\left(-\frac{1}{2}\left(\frac{b}{R}\right)^2\right)-exp\left(-\frac{1}{2}\left(\frac{c}{R}\right)^2\right)\right]\left[Ei\left(-\frac{1}{2}\left(\frac{c}{R}\right)^2\right)-Ei\left(-\frac{1}{2}\left(\frac{b}{R}\right)^2\right)\right]$
- $b \approx 0.59R \qquad and \qquad c = R\left(\sqrt{\frac{\pi}{2}}+\sqrt{\frac{4-\pi}{2}}\right)$



Powanwe and Longtin, Scientific Rep. 2019

Burst duration density (red=theory)

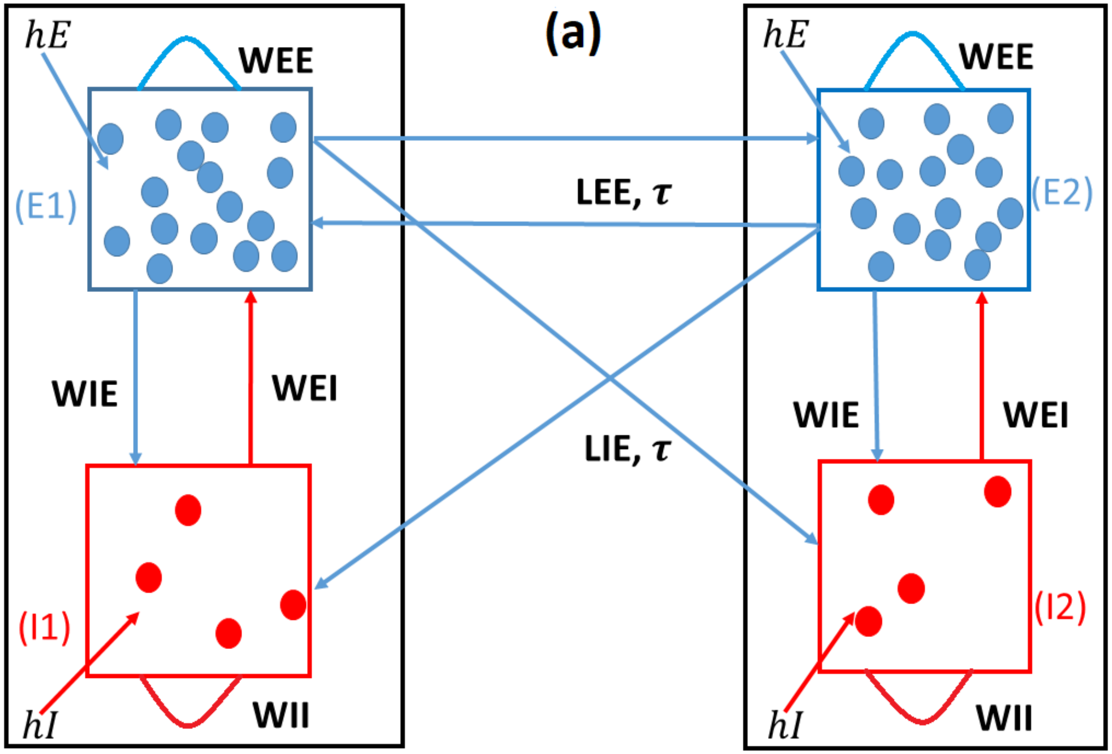


a→b→c→d : approaching the Hopf: burst durations get longer

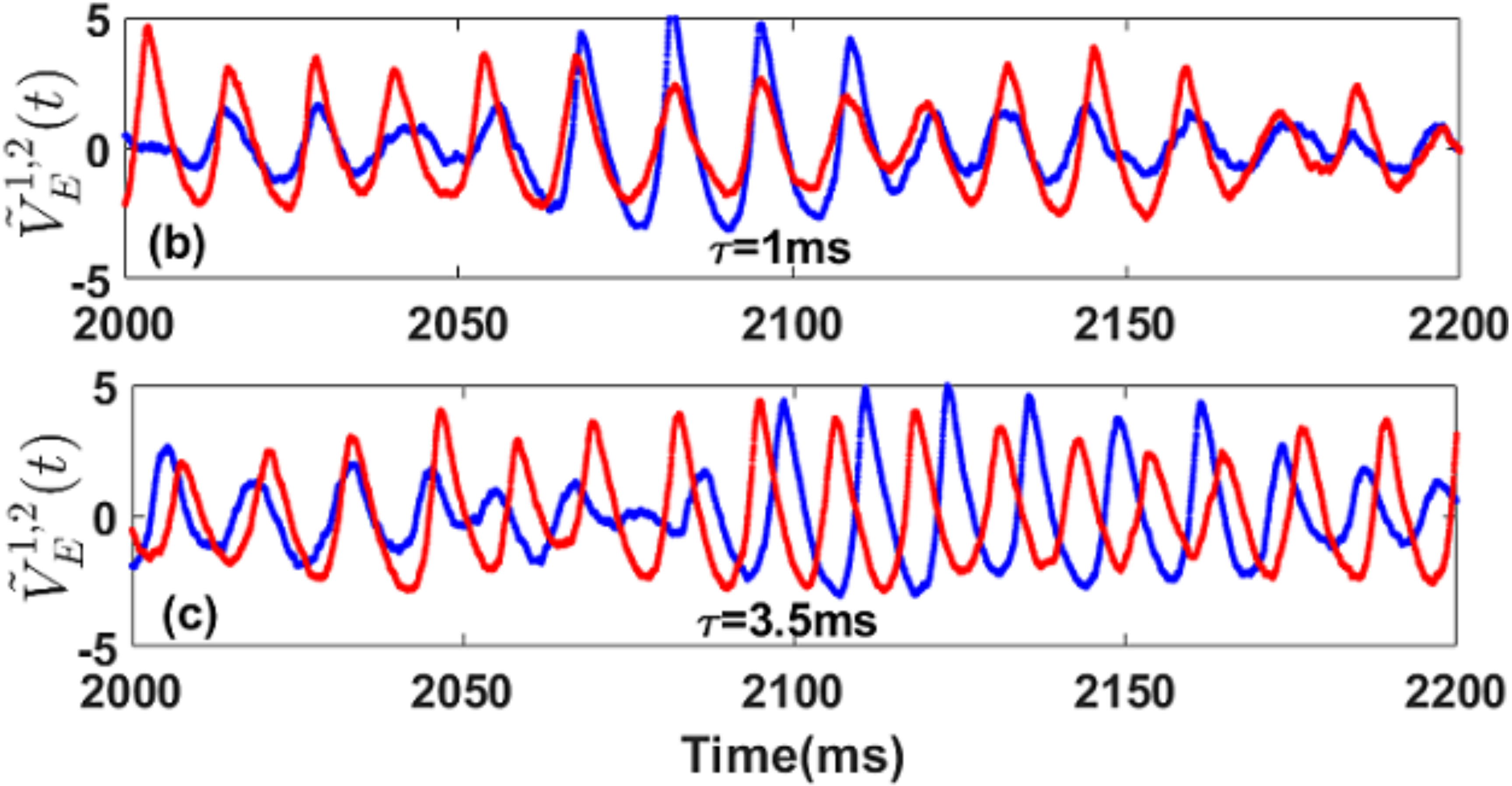
Delay-coupled E-I Networks

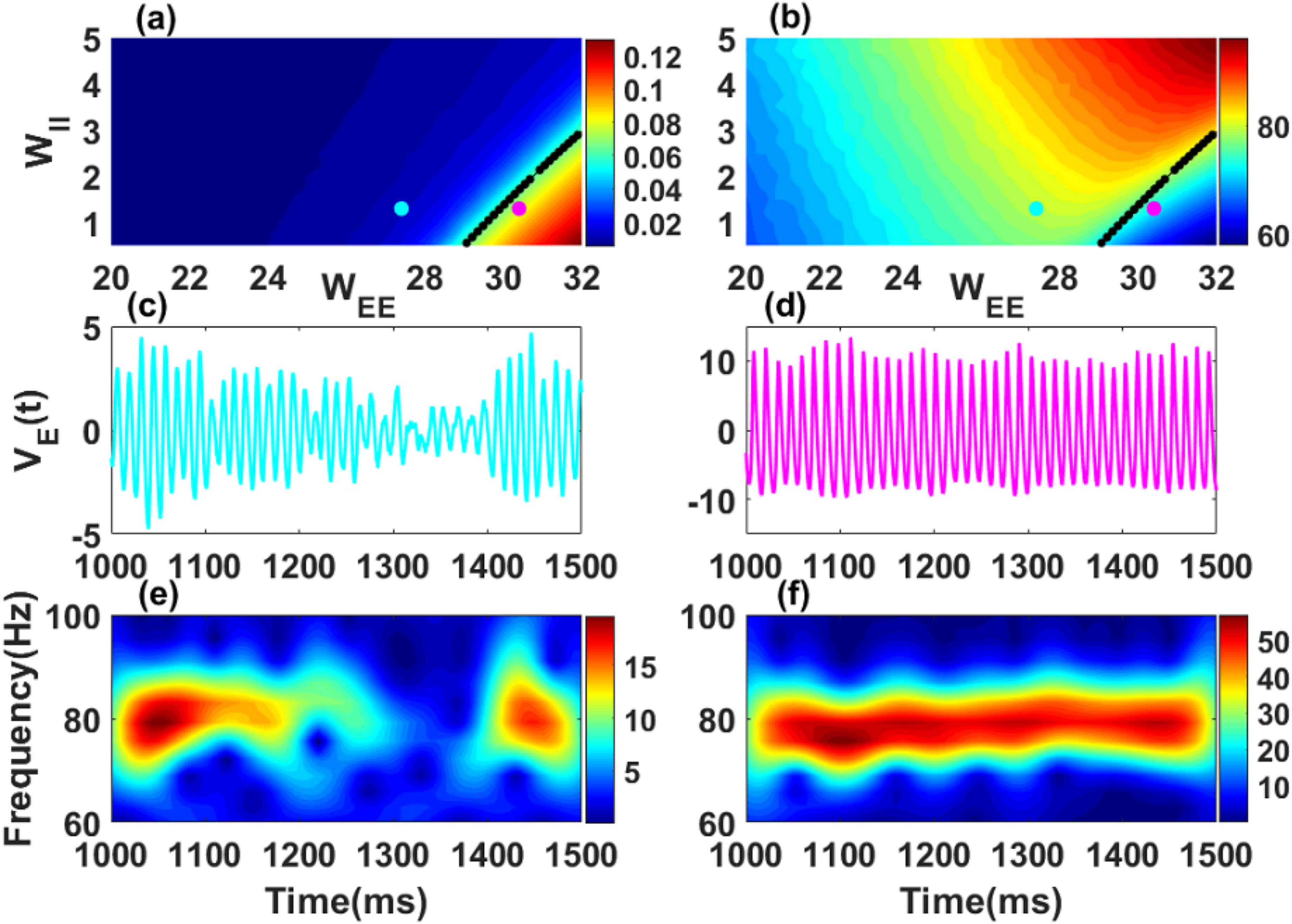
(Powanwe and Longtin, Phys. Rev. Res. 2020)

2 brain areas, each one generating a rhythm.
What happens after coupling?
→ It depends on the delay (and other parameters)



Upper: in-phase
Lower: out-of-phase (0 < Δϕ < π)





fixed point + noise

limit cycle + noise

One theory for both sides of the Hopf bifurcation

Powanwe + Longtin, Phys. Rev. Res. 2021

Wilson-Cowan with nonlinearities looks nastier…

$$\frac{dV_E(t)}{dt} = A_{EE}V_E(t) + A_{EI}V_I(t) + L_{EE}V_E^2(t) + L_{EI}V_E(t)V_I(t) + M_{EI}V_I^2(t) + B_{1E}V_E^3(t) + B_{2E}V_E V_I^2(t) + B_{3E}V_I V_E^2(t) + B_{4E}V_I^3(t) + \eta_E(t) \quad (13)$$

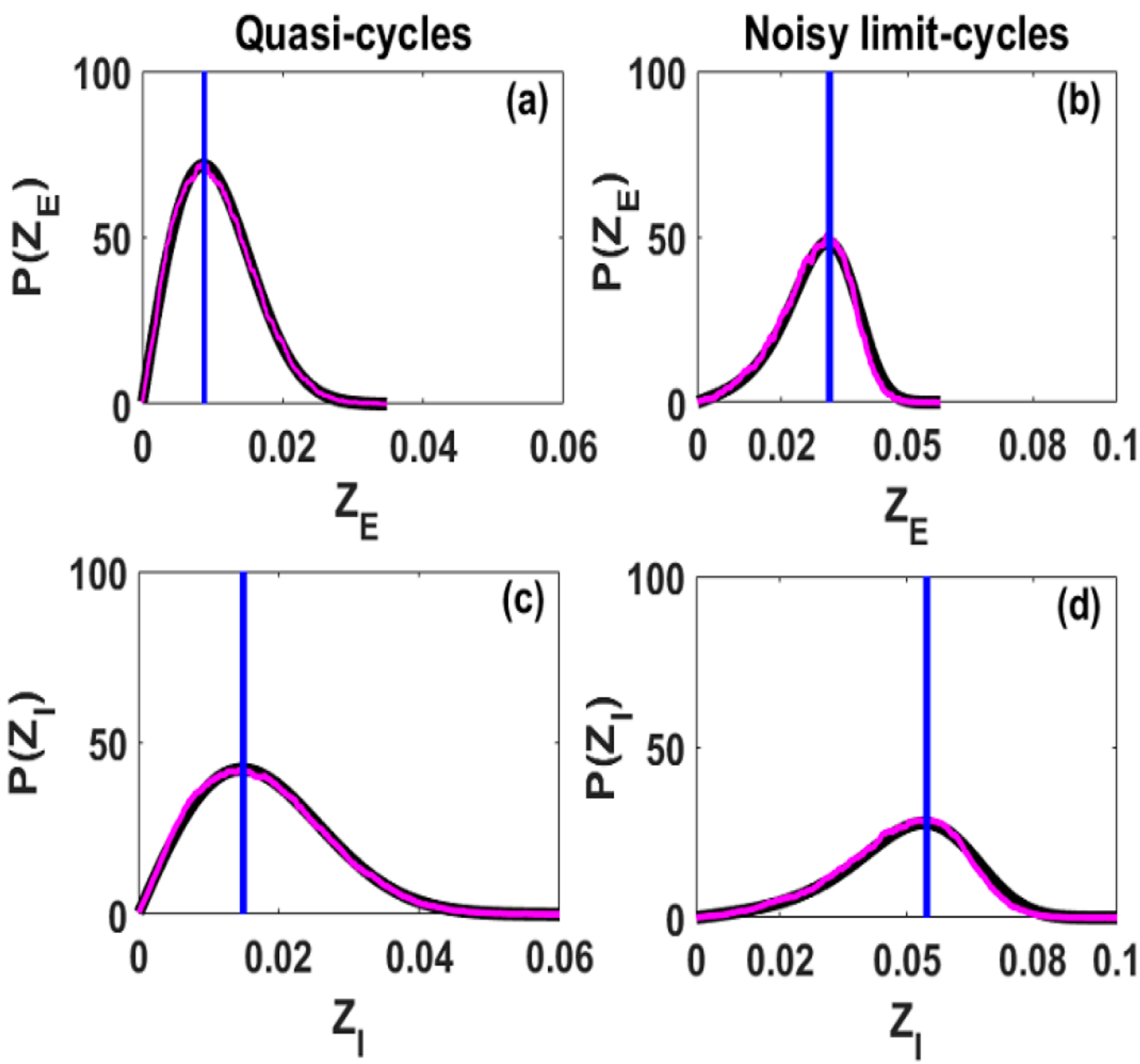
$$\frac{dV_I(t)}{dt} = A_{IE}V_E(t) + A_{II}V_I(t) + L_{II}V_I^2(t) + L_{IE}V_E(t)V_I(t) + M_{IE}V_E^2(t) + B_{1I}V_I^3(t) + B_{2I}V_E V_I^2(t) + B_{3I}V_I V_E^2(t) + B_{4I}V_E^3(t) + \eta_I(t)\,, \quad (14)$$

But envelope-phase dynamics look not so nasty

$$dZ_E = \left[-\nu Z_E + B_1 Z_E^3 + \frac{D}{2Z_E} \right] dt + \sqrt{D} dW_1$$

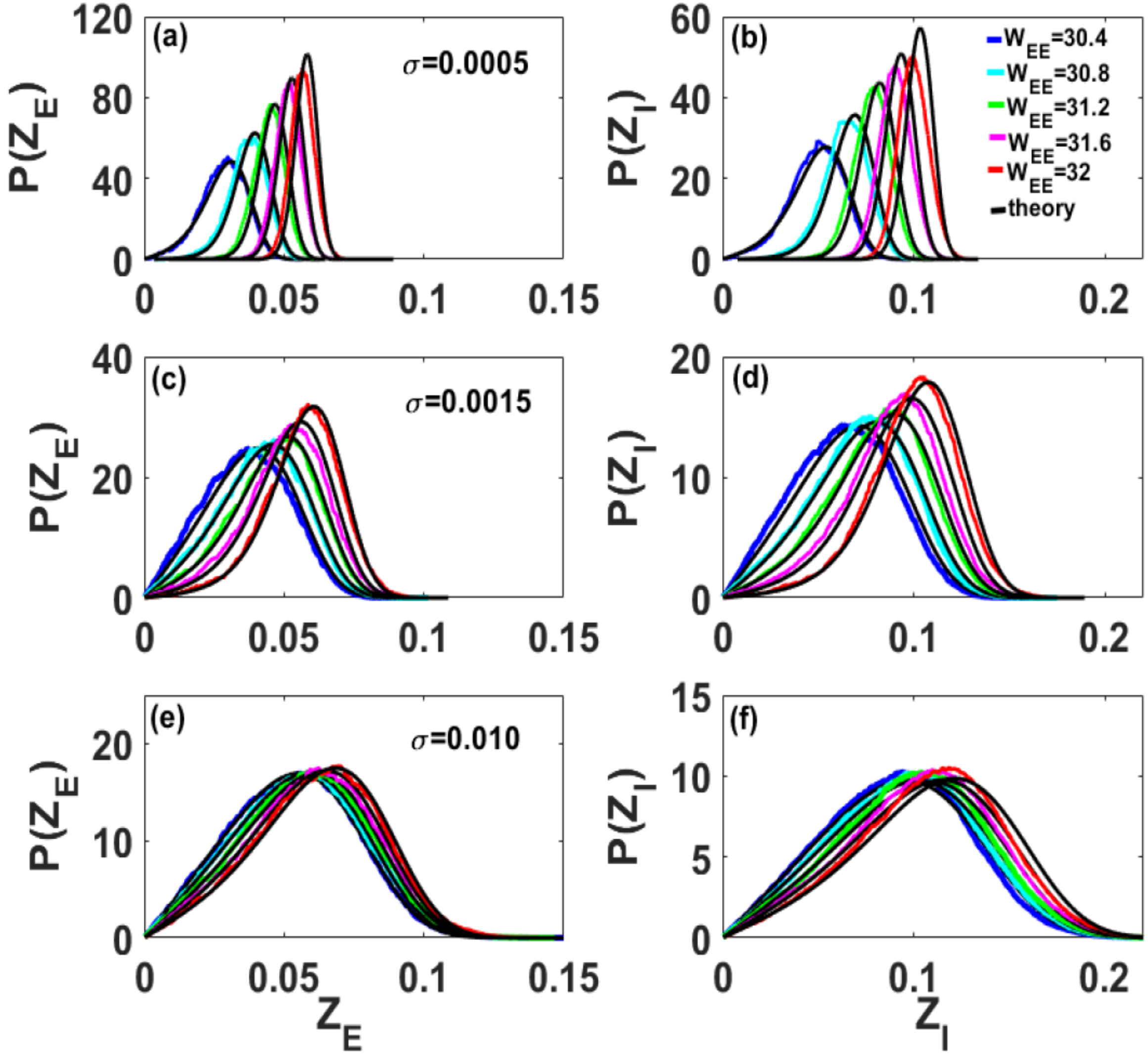
$$d\phi_E = B_2 Z_E^2 \, dt + \frac{\sqrt{D}}{Z_E} dW_2$$

Simulations vs Theory



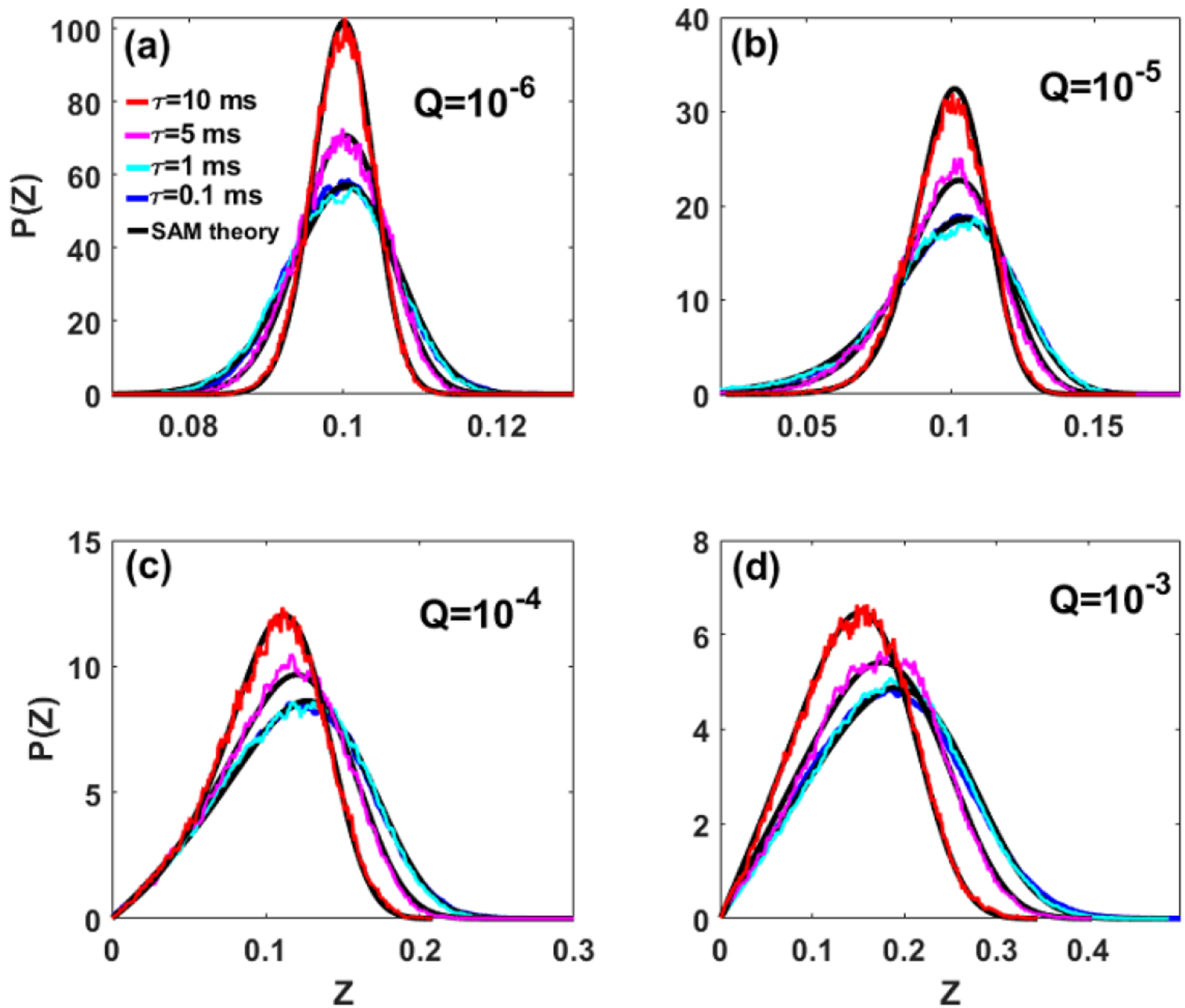
Powanwe + Longtin, Phys. Rev. Res. 2021

Effect of noise intensity on limit cycle amplitude



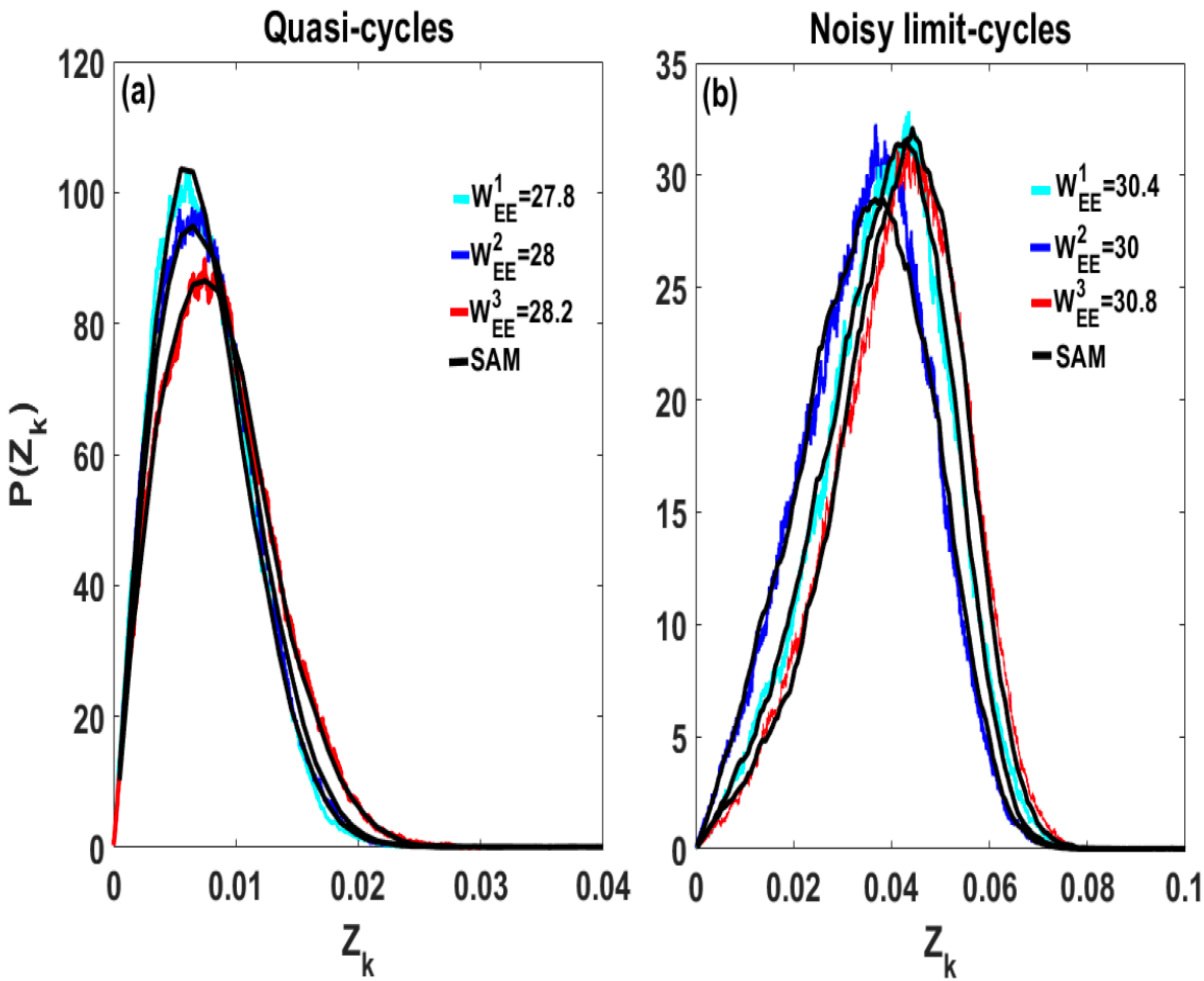
Effect of Noise Correlation Time τ

Stochastic Stuart-Landau model   Q=noise intensity



Heterogeneous Network of 5 coupled Wilson-Cowan systems

Quasi-cycle: Rayleigh density   Limit Cycle: approx. Gaussian



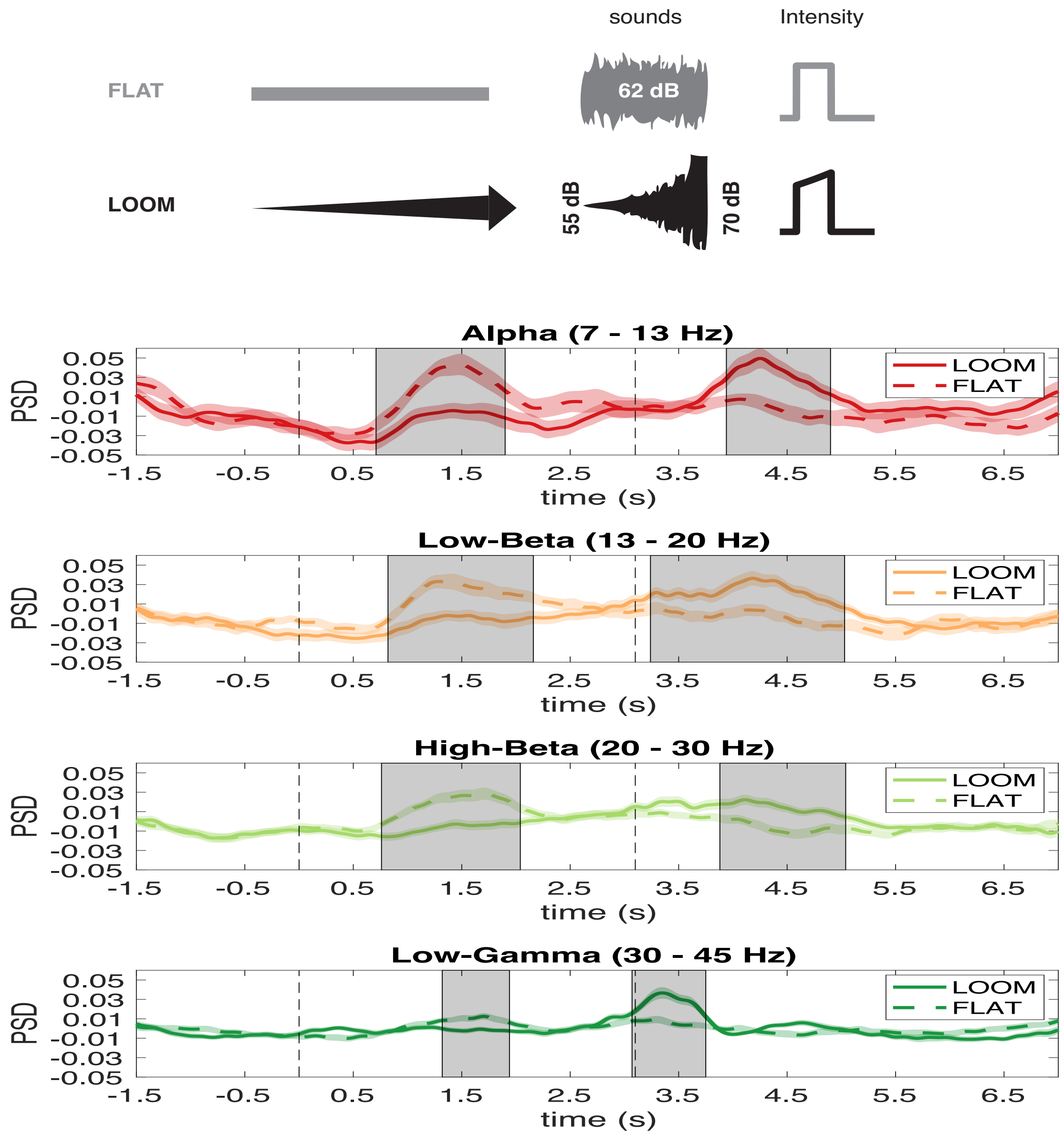
Powanwe + Longtin, Phys. Rev. Res. 2021

Theory of Rhythm Bursts

- Stochastic averaging method to produce envelope-phase equations; one-way coupling to lowest order.
- Accounts for many observed rhythm features
- Identification of an optimal region in parameter space for producing these stochastic dynamics.
- Mean first passage time: burst durations
- Burst duration increases approaching Hopf bifurcation
- Unified description above and below Hopf for coupled brain rhythms
- Help understand information transfer between brain areas
- EXTEND TO OTHER COMPLEX DISTRIBUTED RHYTHMIC PHENOMENA

Question 1: Looming vs flat sound: TEST FOR PERI-PERSONAL SPACE

EEG POWER
vs
TIME



B. Sancristobal, F. Ferri, A. Longtin, MG Perrucci, GL Romani, G. Northoff , Brain Topography 2021

These rhythms are endogeneous
-> periodicity of the sound wave gone after the cochlear nucleus

But the power spectral density "PSD(f)" of these rhythms is modulated by the sound intensity, as well as by the sound onset and offset

Outlook

*Our approach has been extended to coupled networks

→ phase-synchronization/communication between brain areas

*Use the approach to guide <u>therapeutic stimulation</u>

→ theory guides how to correct faulty rhythm statistics

<u>CAVEATS</u>:

* What if network is not simple E-I?
  → can E-I still be a good effective description?

- What about spiking networks?

- What about nonlinearity of f-I curves?

* What about realistic synapses? plasticity?

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ADAM SACHS
The Ottawa Hospital

More Network Physiology

Thoracic surgeon with background in physics

Methods: Nonlinear Time Series Analysis

-fractal dimension
-recurrence plots
-entropies
…

APPLIED TO Heart rate variability, Breathing rate, temperature, blood pressure etc…

-SEPSIS ONSET
-IMPAIRED HEAT METABOLISM
-EXTUBATION OUTCOMES

Andrew Seely
The Ottawa Hospital