Localizing high order effects in time, and across a complex system

SEBINO STRAMAGLIA

Università degli Studi di Bari Aldo Moro & INFN Sezione di Bari

ISINP 2022– Como July 2022

Summary

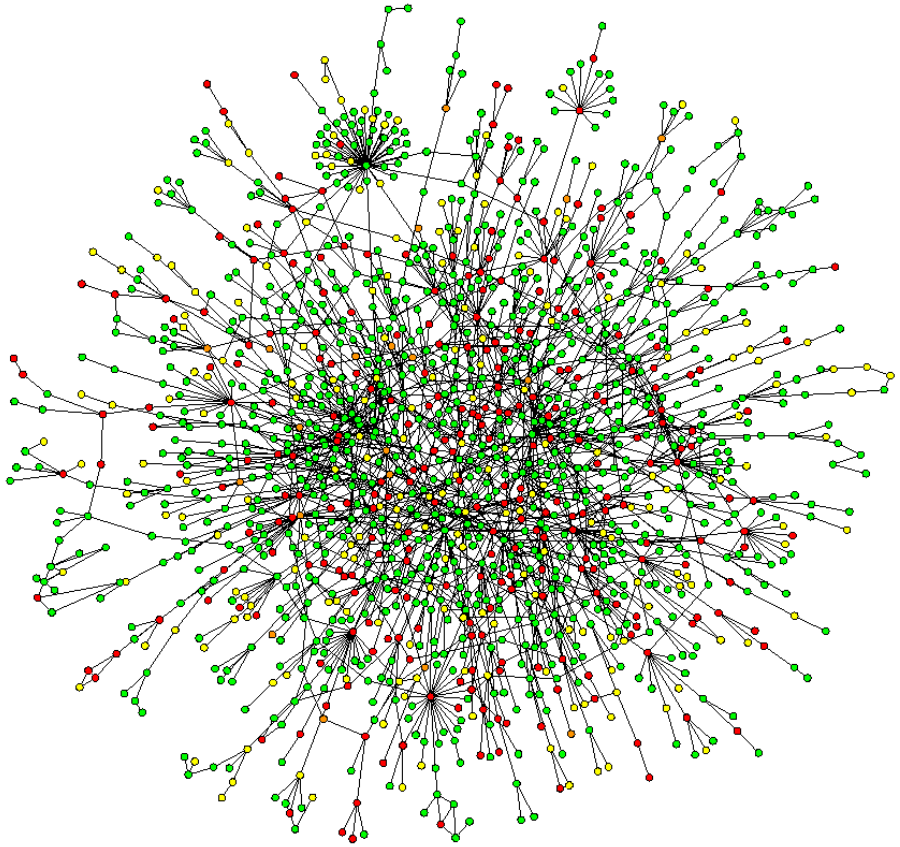
1) O-information: spiking neurons

2) Informational character of patterns: application in music

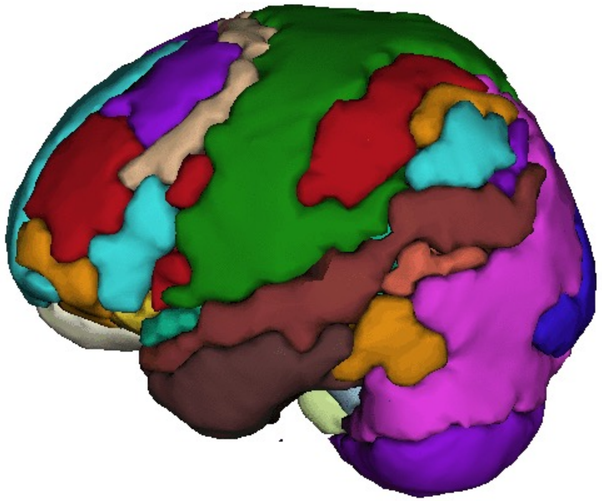
3) Gradients of O-information

4) Conclusions

Complex Networks



Network Physiology, Network Neuroscience, Network Psychiatry, etc…



Functional Segregation vs Functional Integration

nature physics

PERSPECTIVE

https://doi.org/10.1038/s41567-021-01371-4

The physics of higher-order interactions in complex systems

Federico Battiston[1] ✉, Enrico Amico[2,3], Alain Barrat[4,5], Ginestra Bianconi[6,7], Guilherme Ferraz de Arruda[8], Benedetta Franceschiello[9,10], Iacopo Iacopini[1], Sonia Kéfi[11,12], Vito Latora[6,13,14,15], Yamir Moreno[8,15,16,17], Micah M. Murray[9,10,18], Tiago P. Peixoto[1,19], Francesco Vaccarino[20] and Giovanni Petri[8,21] ✉

Complex networks have become the main paradigm for modelling the dynamics of interacting systems. However, networks are intrinsically limited to describing pairwise interactions, whereas real-world systems are often characterized by higher-order interactions involving groups of three or more units. Higher-order structures, such as hypergraphs and simplicial complexes, are therefore a better tool to map the real organization of many social, biological and man-made systems. Here, we highlight recent evidence of collective behaviours induced by higher-order interactions, and we outline three key challenges for the physics of higher-order systems.

Correspondence | Published: 21 March 2022

Disentangling high-order mechanisms and high-order behaviours in complex systems

Fernando E. Rosas, Pedro A. M. Mediano, Andrea I. Luppi, Thomas F. Varley, Joseph T. Lizier, Sebastiano Stramaglia, Henrik J. Jensen & Daniele Marinazzo

Nature Physics **18**, 476–477 (2022) | Cite this article

High Order Mechanisms

-Structure

-Interactions

How the system is structured

High Order Behaviours

-Function (correlations)

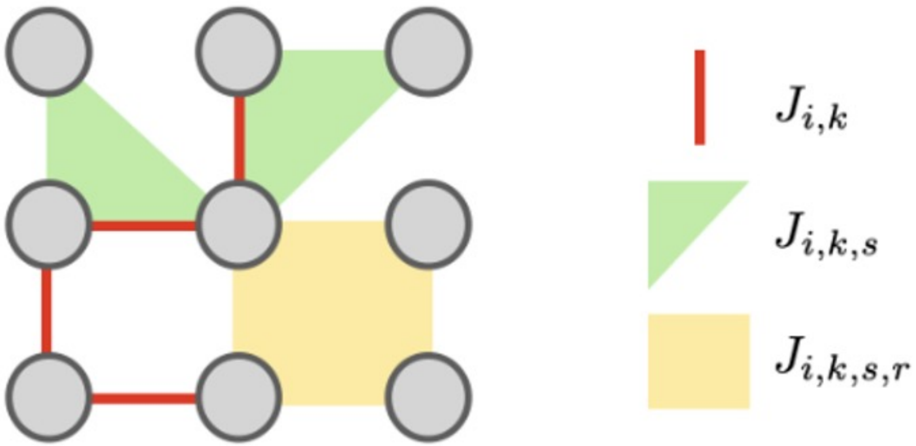
-Observables (from data)

What the system does

A)

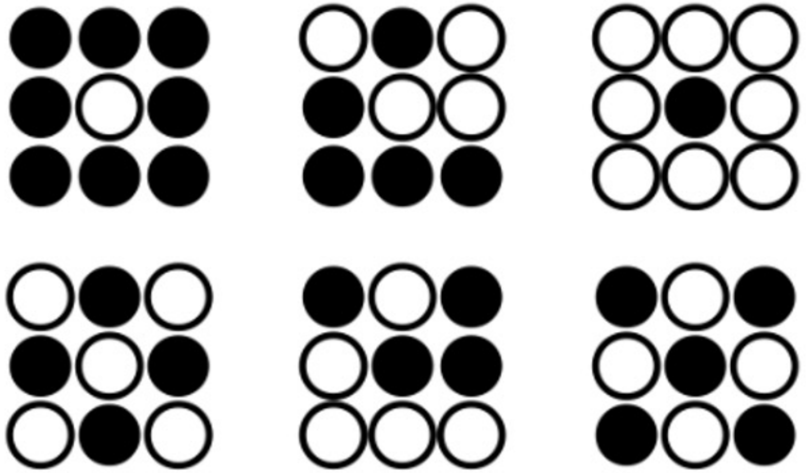
Mechanism

$$\mathcal{H}(x_1, \ldots, x_n) = -\sum_{i,k} J_{i,k} x_i x_k \ldots - J_{1,\ldots,n} \prod_{s} x_s$$

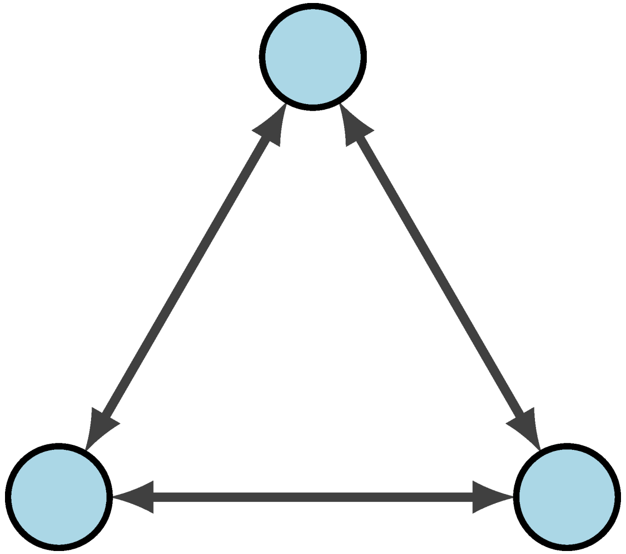


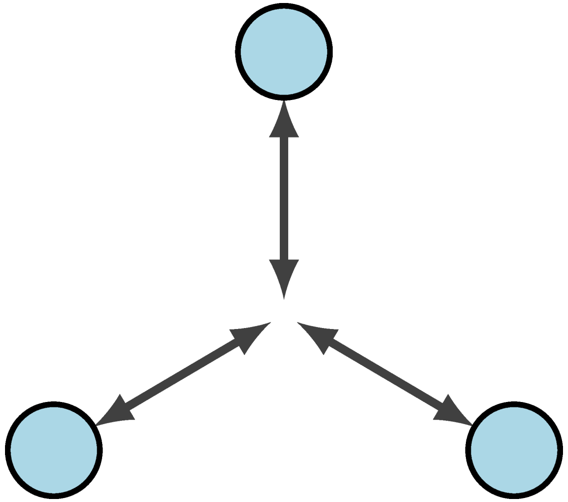
Observed behaviour

$$p(x_1, \ldots, x_n) = \frac{e^{-\beta \mathcal{H}(x_1, \ldots, x_n)}}{Z}$$



There are two basic types of high-order dependencies





redundancy!

synergy!

Beyond triplets:

O-information: useful tool for practical data analysis, to assess the informational character of multiplets of variables

$$\mathrm{TC}(\boldsymbol{X}^n) = \sum_{i=1}^{n} H(X_i) - H(\boldsymbol{X}^n)$$

Total Correlation (Redundancy)

$$\mathrm{DTC}(\boldsymbol{X}^n) = H(\boldsymbol{X}^n) - \sum_{i=1}^{n} H(X_i \mid \boldsymbol{X}^n_{-i})$$

Dual Total Correlation (Synergy)

$$\Omega_n := \mathrm{TC}(\boldsymbol{X}^n) - \mathrm{DTC}(\boldsymbol{X}^n)$$

Captures the balance between Redundancy and Synergy

O-Information

$$\Omega_n = (n-2)H(\boldsymbol{X}^n) + \sum_{j=1}^{n} [H(X_j) - H(\boldsymbol{X}^n | X_j)]$$

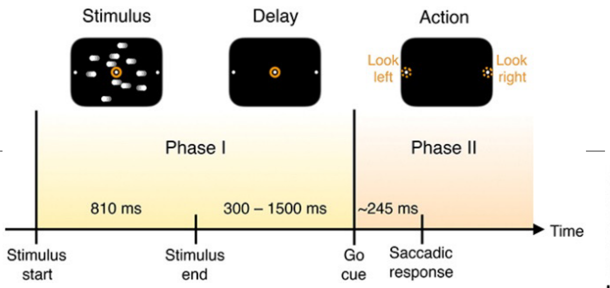
$\Omega_n > 0$ Redundancy

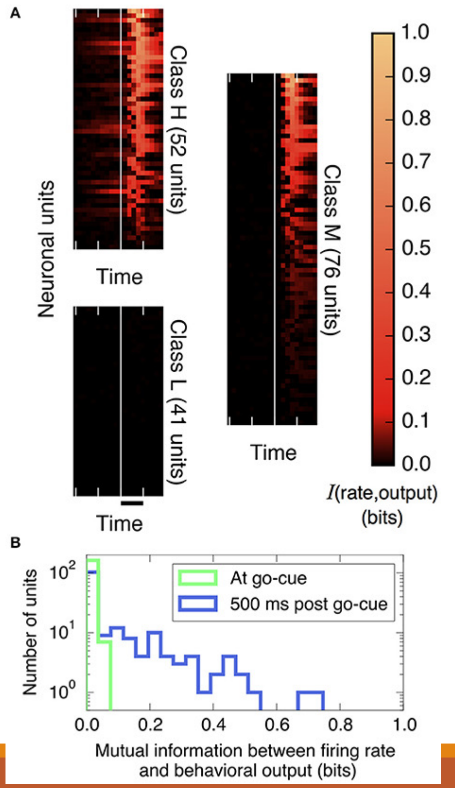
$\Omega_n < 0$ Synergy

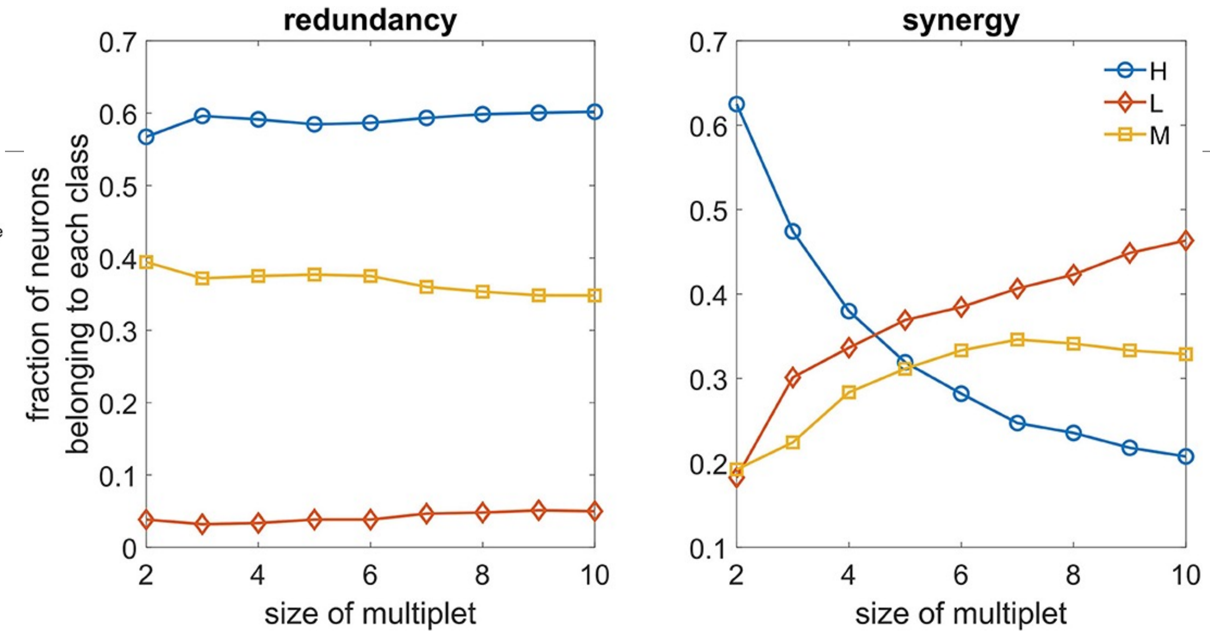
Quantifying high-order interdependencies via multivariate extensions of the mutual information

Fernando E. Rosas, Pedro A. M. Mediano, Michael Gastpar, and Henrik J. Jensen
Phys. Rev. E **100**, 032305 – Published 13 September 2019

APPLICATION TO SPIKING NEURONS







Front. Neurosci., 06 June 2017 | https://doi.org/10.3389/fnins.2017.00313

Dual Coding Theory Explains Biphasic Collective Computation in Neural Decision-Making

Bryan C. Daniels[1*], Jessica C. Flack[1,2] and David C. Krakauer[1,2]

Front. Physiol., 14 January 2021 | https://doi.org/10.3389/fphys.2020.595736

Quantifying Dynamical High-Order Interdependencies From the O-Information: An Application to Neural Spiking Dynamics

Sebastiano Stramaglia[1,2*], Tomas Scagliarini[1], Bryan C. Daniels[3] and Daniele Marinazzo[4]

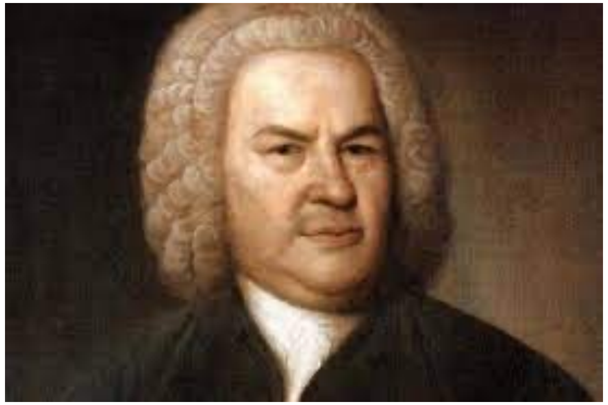
Quantifying high-order interdependencies on individual patterns via the local O-information

$$Entropy = -\int \log\big(p(x)\big)\, p(x)dx$$

$$Local\ Entropy = -\log\big(p(x)\big)\,;\ surprise$$

Local O-information is obtained from the O-information substituting the entropy with the local entropy

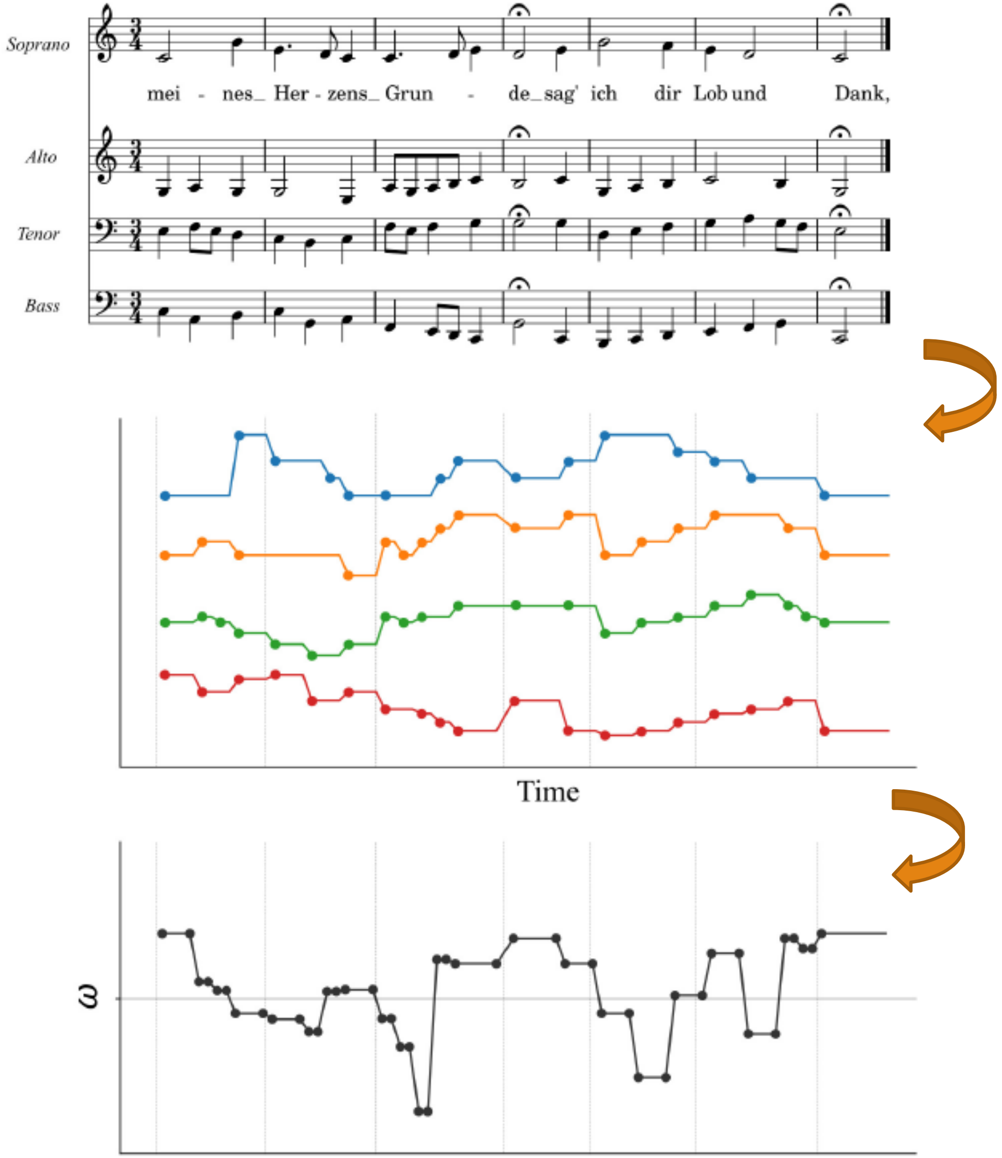
Scagliarini et al.: Phys. Rev. Res.



J.S. Bach (1685-1750)

- 172 chorales
- $\sim 40 * 10^3$ 4-note chords

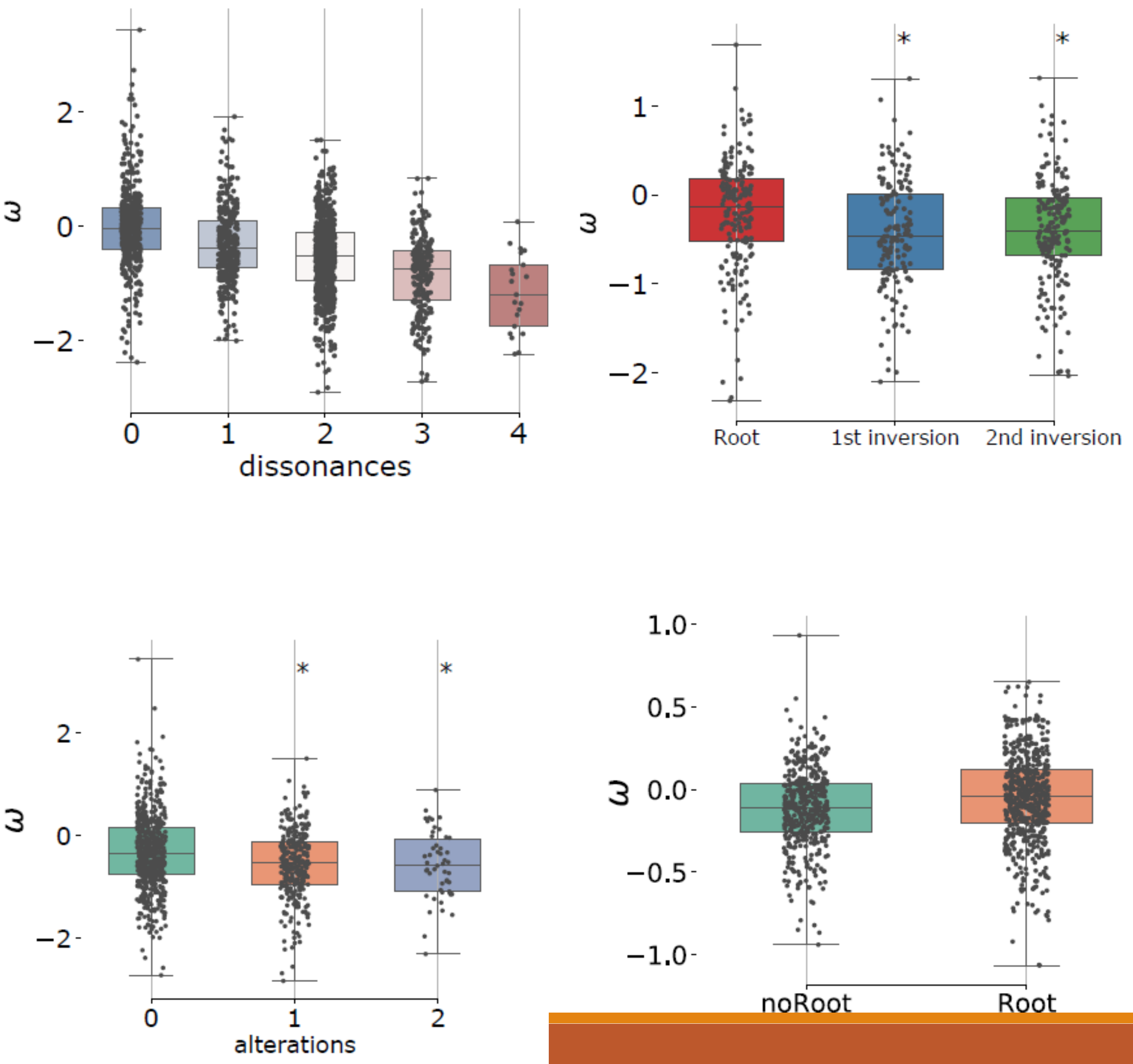
$$p(x_1, x_2, x_3, x_4)$$



Music complexity

Synergy!

Redundancy		Synergy	
Chord	ω	**Chord**	ω
R R R R	3.443	A E D D	-2.916
G D G D	2.736	G B F♯ E	-2.836
F C F C	2.484	B F B B	-2.725
A C A C	2.311	A♯ E E A	-2.688
C G C C	2.23	G F♯ F♯ A	-2.613
E G E G	2.228	G C B A	-2.581
C G C G	2.127	F A♯ G F	-2.559
A A E A	1.93	G C C A♯	-2.522
F D G D	1.921	G E C♯ A	-2.432
D D A A	1.824	G G G♯ C	-2.396
G D G G	1.782	R G R E	-2.388
D D A D	1.748	G♯ F G♯ C	-2.311
D F C A	1.688	A♯ F G♯ C	-2.276
G G D G	1.674	G A F G	-2.245
F F C F	1.594	E G A F	-2.238
E C E C	1.586	E F♯ C D	-2.221
A C A D	1.544	F♯ F♯ C♯ A	-2.219
F F C D	1.532	G F F A♯	-2.185
R R R A	1.522	E A G D	-2.176
G F G D	1.512	C♯ G G B	-2.173



O-information rate (L. Faes et al.): a framework which allows decomposing in frequency

A Framework for the Time- and Frequency-Domain Assessment of High-Order Interactions in Brain and Physiological Networks

Luca Faes,[1,*] Gorana Mijatovic,[2] Yuri Antonacci,[3] Riccardo Pernice,[4] Chiara Barà,[4] Laura Sparacino,[4] Marco Sammartino,[4] Alberto Porta,[5] Daniele Marinazzo,[6] and Sebastiano Stramaglia[7]

[1] *Department of Engineering, University of Palermo, Italy*
[2] *Faculty of Technical Sciences, University of Novi Sad, Serbia*
[3] *Department of Physics and Chemistry "Emilio Segrè", University of Palermo, Italy*
[4] *Department of Engineering, University of Palermo, Italy*
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[6] *Department of Data Analysis, University of Ghent, Belgium*
[7] *Department of Physics, University of Bari Aldo Moro, and INFN Sezione di Bari, Italy*

(Dated: February 10, 2022)

Gradients of O-information: low-order descriptors of high-order dependencies

$$\partial_i \Omega(\boldsymbol{X}^n) = \Omega(\boldsymbol{X}^n) - \Omega(\boldsymbol{X}^n_{-i})$$

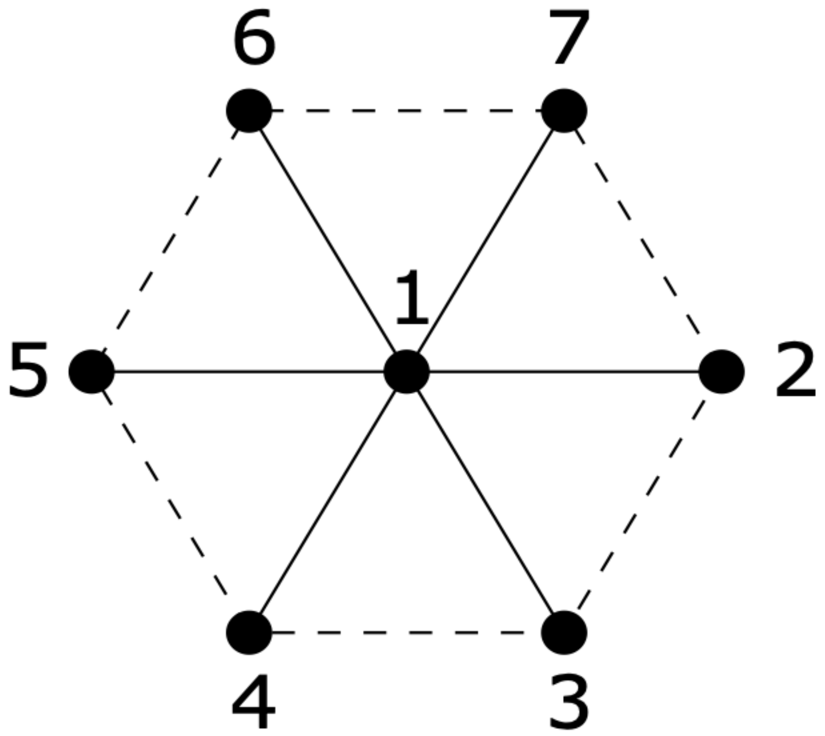
$$\partial_j \partial_i \Omega(\boldsymbol{X}^n) = \partial_i \Omega(\boldsymbol{X}^n) - \partial_i \Omega(\boldsymbol{X}^n_{-j})$$

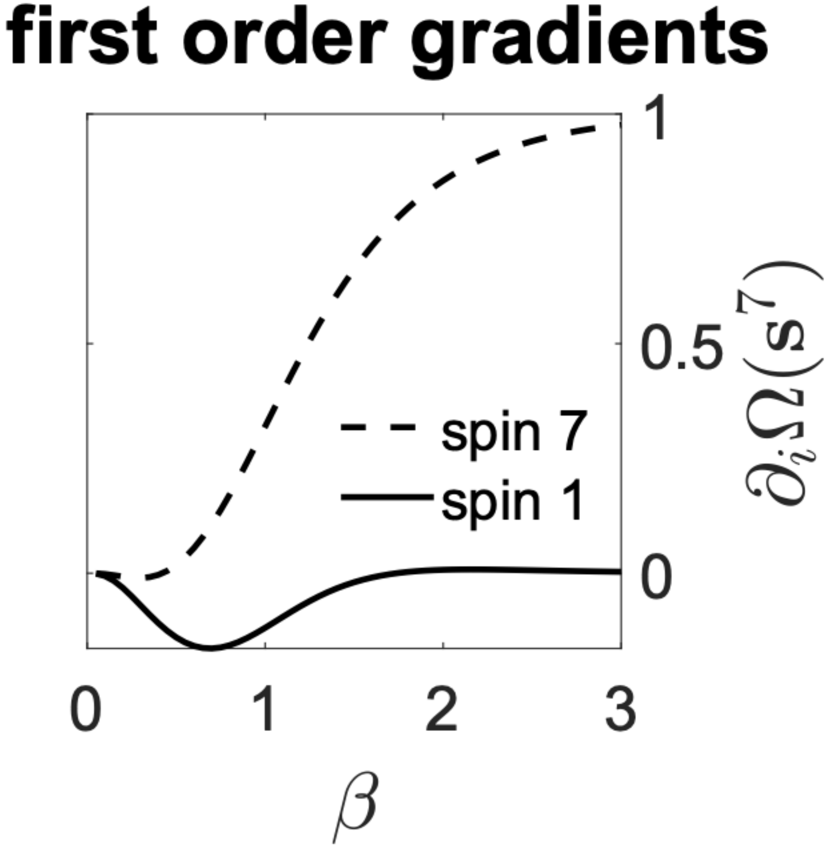
$$\begin{aligned}\partial^2_{ij} \Omega(\boldsymbol{X}^n) = & \left[\Omega(\boldsymbol{X}^n) - \Omega(\boldsymbol{X}^n_{-ij})\right] \\ & - \left[\Omega(\boldsymbol{X}^n_{-i}) - \Omega(\boldsymbol{X}^n_{-ij})\right] - \left[\Omega(\boldsymbol{X}^n_{-j}) - \Omega(\boldsymbol{X}^n_{-ij})\right]\end{aligned}$$

$$\partial_{\gamma}^{|\gamma|}\Omega(\boldsymbol{X}^n) = \sum_{\alpha \subseteq \gamma} (-1)^{|\alpha|}\Omega(\boldsymbol{X}^n{}_{-\alpha})$$

$$\begin{aligned}\partial_{ijk}^{3}\Omega(\boldsymbol{X}^n) = \Omega(\boldsymbol{X}^n) - \Omega(\boldsymbol{X}^n{}_{-i}) - \Omega(\boldsymbol{X}^n{}_{-j}) - \Omega(\boldsymbol{X}^n{}_{-k}) \\ + \Omega(\boldsymbol{X}^n{}_{-ij}) + \Omega(\boldsymbol{X}^n{}_{-ik}) + \Omega(\boldsymbol{X}^n{}_{-jk}) - \Omega(\boldsymbol{X}^n{}_{-ijk})\end{aligned}$$

Toy model: Ising spins





As an econometric application, 14 US macroeconomic time series taken from the Federal Reserve Economic Dataset over a period of 61 years (1959- 2020)

US macroeconomics indicators	$\partial_i \Omega$
COE	0.59
HOANBS	0.47
GDPDEF	0.33
UNRATE	0.27
FEDFUNDS	0.15
TB3MS	0.11
M2SL	0.09
GPDI	−0.26

TABLE I: Gradients of O-information for US macroeconomic indicators (only statistically significant values).

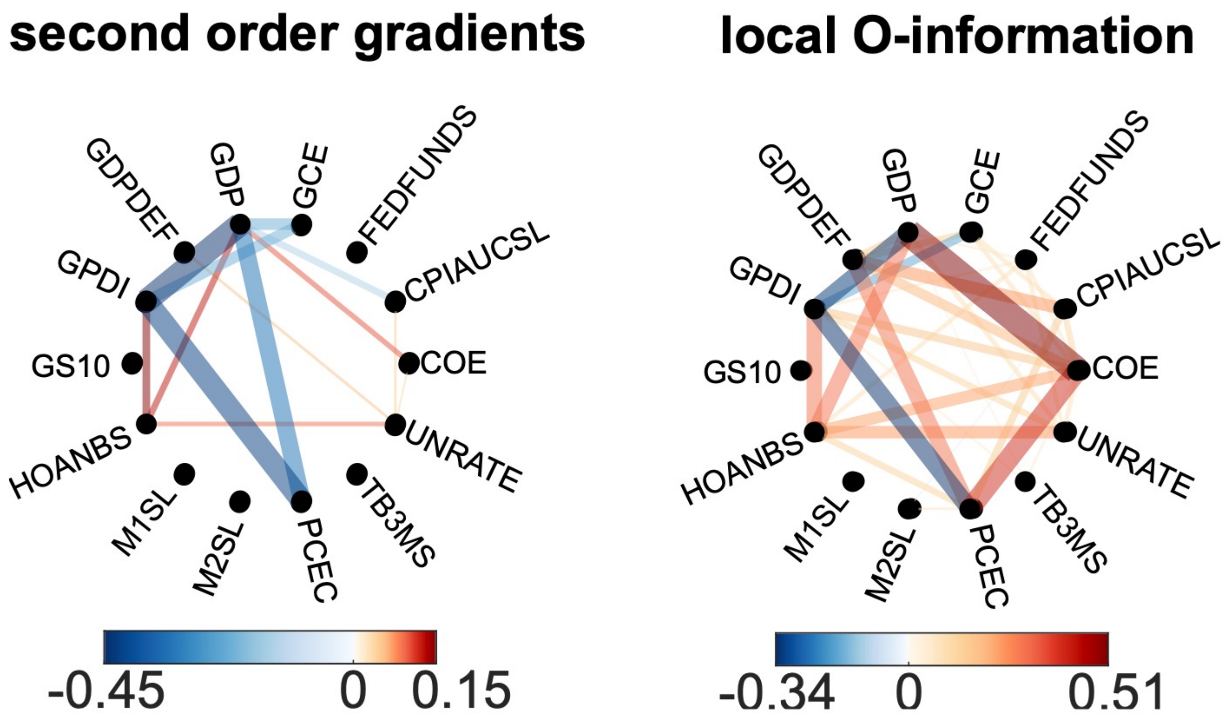
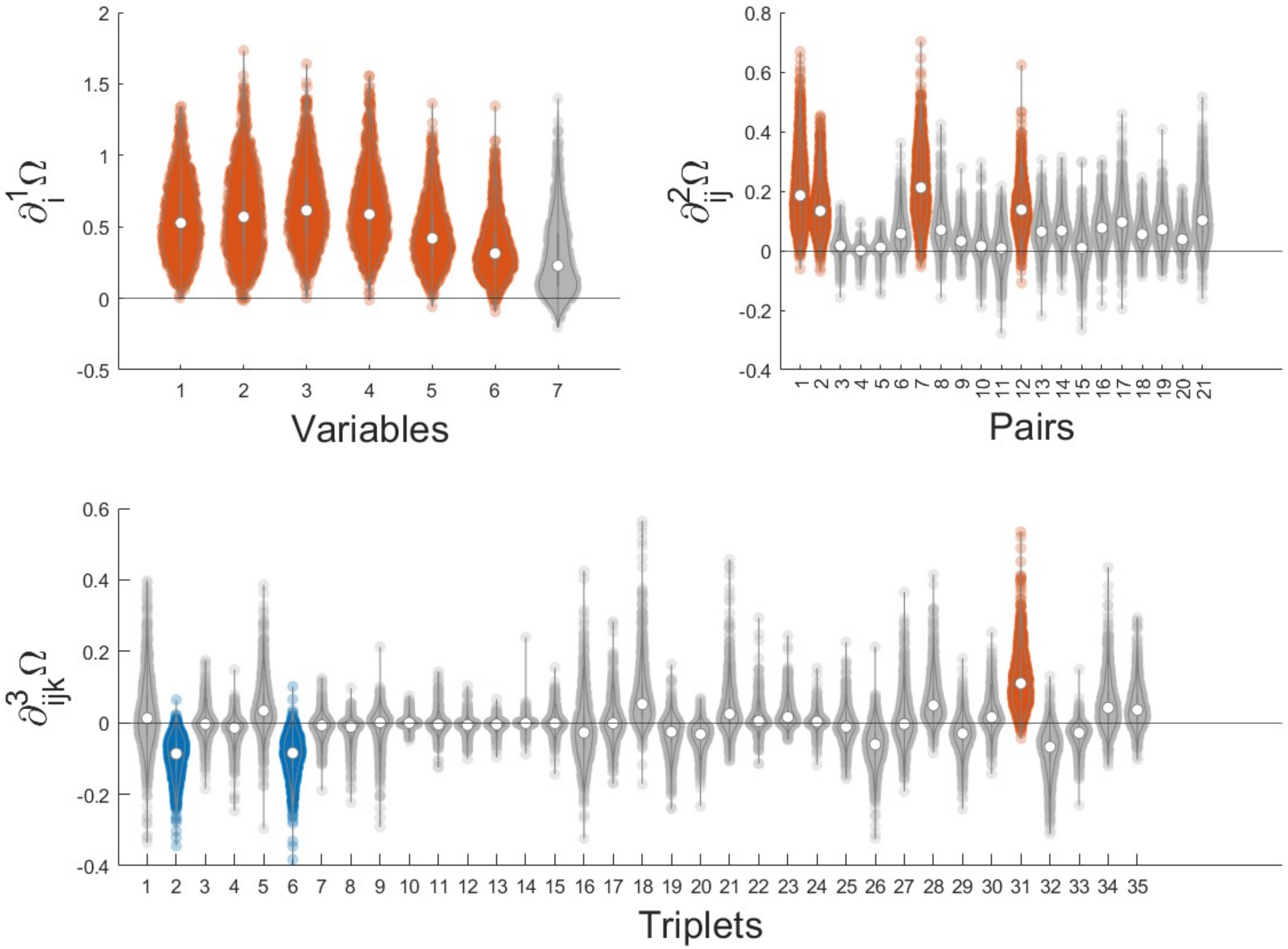


FIG. 2: **Left**: second order gradients for pairs of economic indicators. **Right**: local O-information of pairs of economic indicators. Edge values are encoded by color (sign) and width (absolute value). Only statistically significant edges — calculated via bootstrap resampling — are included.

Application to fMRI data

- 7 series obtained from 100 series averaged over the seven intrinsic connectivity network [1]
- 1083 healthy subjects



Two synergetic triplets at order 3:
{1,2,4} {1,3,4}

[1] Yeo, B. T.; Krienen, F. M.; Sepulcre, J.; Sabuncu, M. R.; Lashkari, D.; Hollinshead, M.; Roffman, J. L.; Smoller, J. W.; Zöllei, L.; Polimeni, J. R. The Organization of the Human Cerebral Cortex Estimated by Intrinsic Functional Connectivity. *Journal of neurophysiology* 2011.

Conclusions

These new tools make possible the analysis of many body effects in complex systems with a computational burden which scales gracefully with the number of variables. The search for synergistic informational circuits can thus be acomplished also in the big data scenario.

Thanks to:

- Tomas Scagliarini, Davide Nuzzi (Bari U)
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- Alberto Porta (Milano U)
- Gorana Mijatovic (Novi Sad U. Serbia)
- Bryan Daniels (Arizona State U)