Modelling High-Order Behaviours in Network Physiology using Information Theory

SEBINO STRAMAGLIA

Università degli Studi di Bari Aldo Moro & INFN Sezione di Bari

ISINP 2022– Como July 2022

Summary

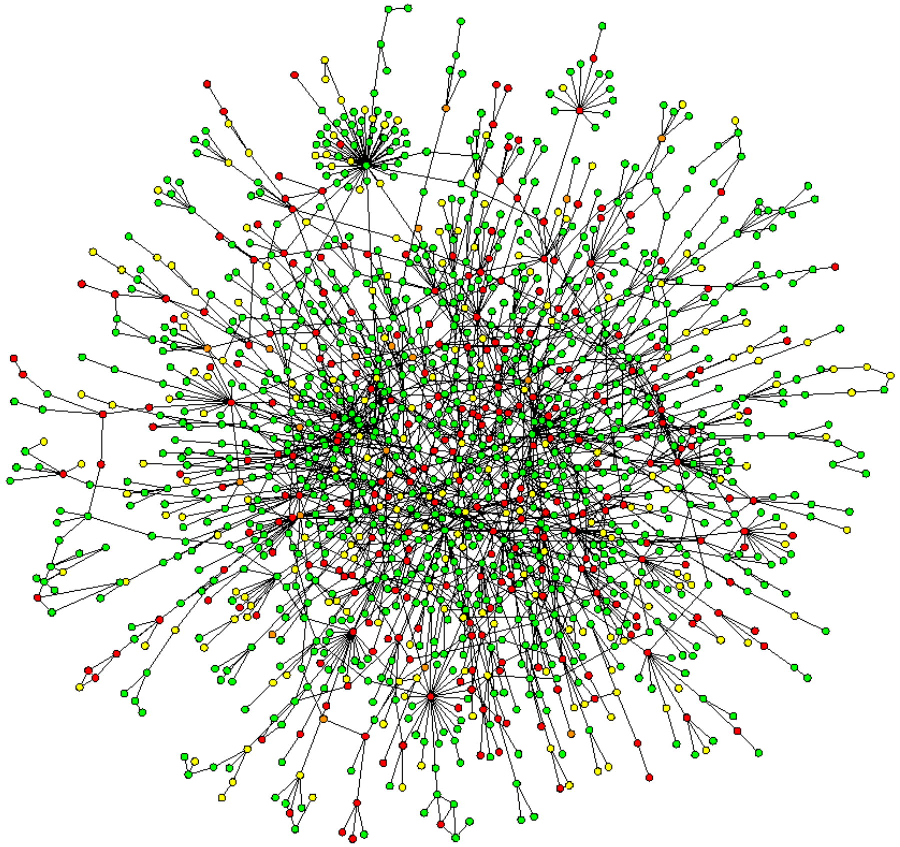
1) High Order Mechanisms vs High Order Behaviors

2) Partial Information Decomposition: synergy and redundancy

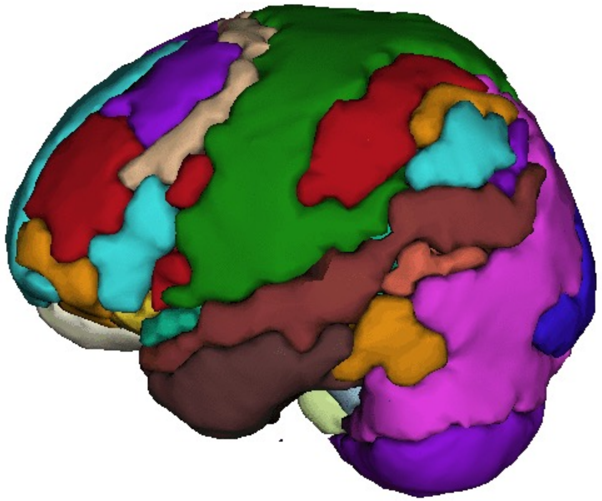
3) Applications

4) Conclusions

Complex Networks



Network Physiology, Network Neuroscience, Network Psychiatry, etc…



Functional Segregation vs Functional Integration

nature physics

PERSPECTIVE

https://doi.org/10.1038/s41567-021-01371-4

Check for updates

The physics of higher-order interactions in complex systems

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Complex networks have become the main paradigm for modelling the dynamics of interacting systems. However, networks are intrinsically limited to describing pairwise interactions, whereas real-world systems are often characterized by higher-order interactions involving groups of three or more units. Higher-order structures, such as hypergraphs and simplicial complexes, are therefore a better tool to map the real organization of many social, biological and man-made systems. Here, we highlight recent evidence of collective behaviours induced by higher-order interactions, and we outline three key challenges for the physics of higher-order systems.

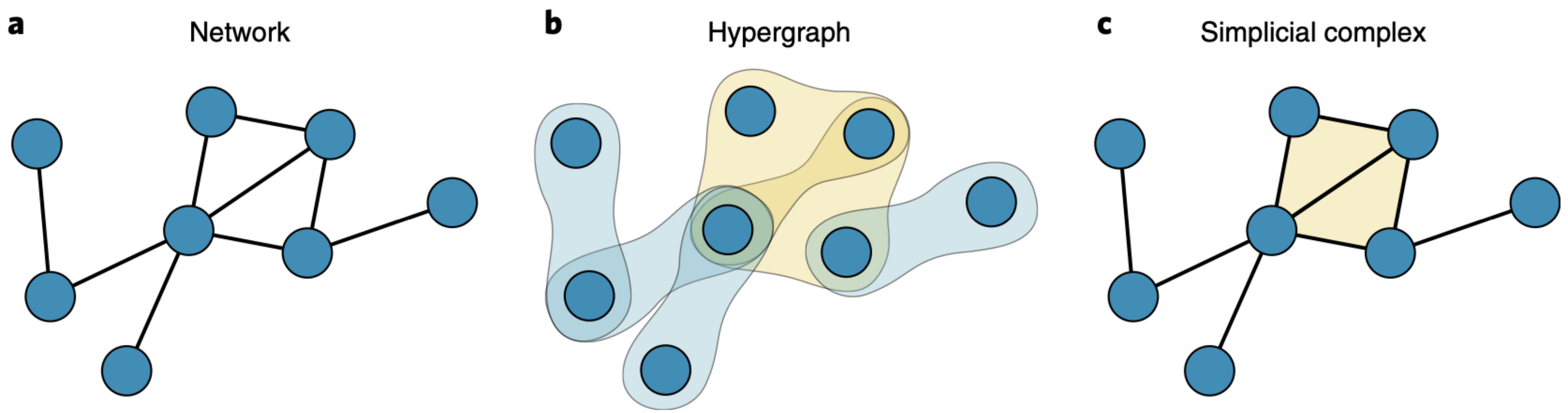


Fig. 1 | Pairwise and higher-order representations. a, Systems comprising many interacting units have long been represented as networks, with interactions restricted to pairs of nodes and represented as edges. However, it is not always possible to describe group interactions as sums of pairwise interactions only. **b**, Representations allowing for genuine group interactions include hypergraphs, which can encode interactions among an arbitrary number of units without further constraints. Here, shaded groups of nodes represent hyperedges. **c**, Simplicial complexes offer another approach. Although more constrained than hypergraphs, they provide access to powerful mathematical formalisms[111]. Edges (1-simplices) are shown here in black, full triangles (2-simplices) in yellow. Note that, in simplicial complexes, all subfaces of a simplex (for example, the edges of a triangle) need to be included. This constraint does not hold for hypergraphs.

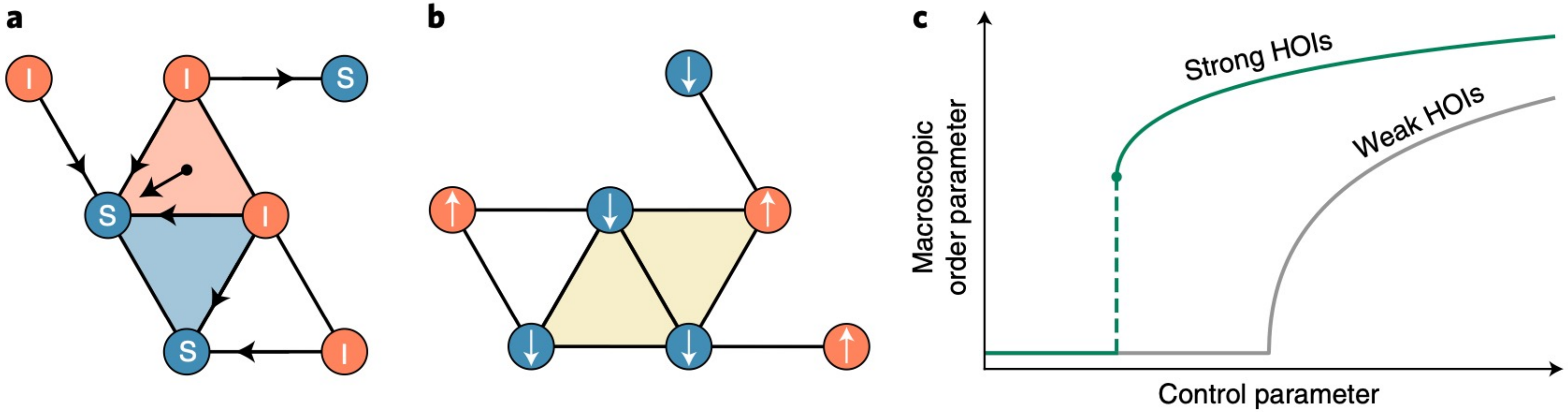


Fig. 2 | Higher-order interactions lead to explosive phenomena. Edges and hyperedges encode pairwise and group-level couplings among the nodes of a complex system. **a**, Hyperedges modulate group infection and many-body feedback in higher-order processes of contagion. Susceptible nodes (S, blue) can be infected by infectious ones (I, orange) in the usual way along edges, but also by groups containing a large fraction of infected nodes (for example, orange 2-simplices). **b**, Hyperedges have a similar effect on higher-order processes of synchronization, in which oscillators on nodes can be coupled along edges, or in groups via higher-order interactions (HOIs). **c**, Abrupt transitions emerge when increasing the strength of such interactions, suggesting a general pathway to explosive phenomena.

Correspondence | Published: 21 March 2022

Disentangling high-order mechanisms and high-order behaviours in complex systems

Fernando E. Rosas, Pedro A. M. Mediano, Andrea I. Luppi, Thomas F. Varley, Joseph T. Lizier, Sebastiano Stramaglia, Henrik J. Jensen & Daniele Marinazzo

Nature Physics **18**, 476–477 (2022) | Cite this article

High Order Mechanisms

-Structure
-Interactions

How the system is structured

High Order Behaviours

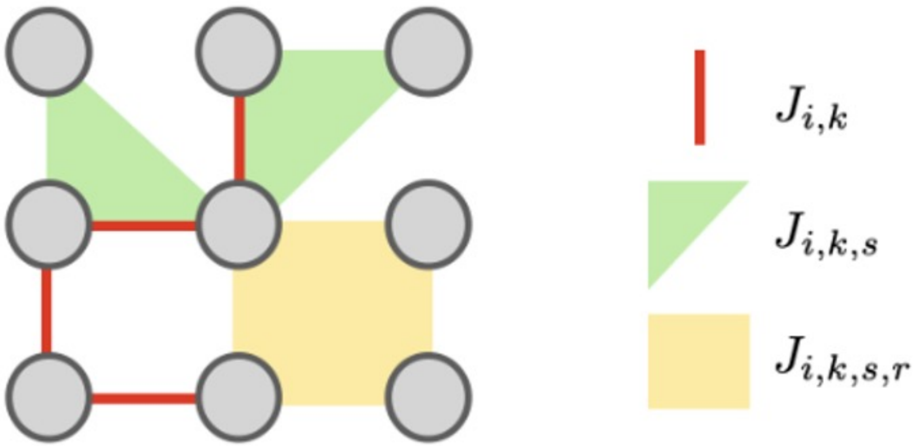
-Function (correlations)

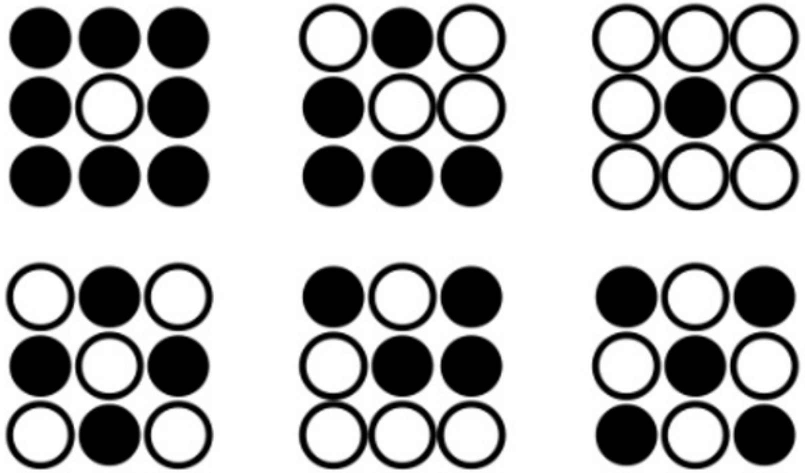
-Observables (from data)

What the system does

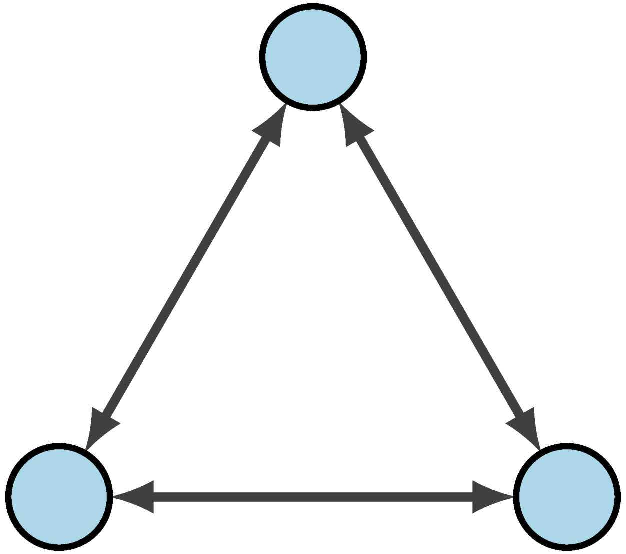
A) **Mechanism** **Observed behaviour**

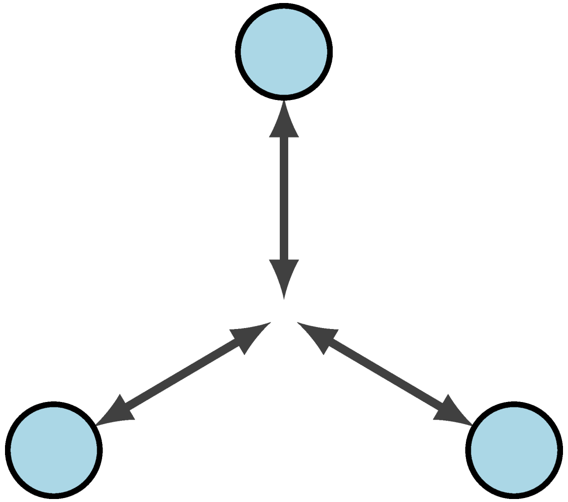
$$\mathcal{H}(x_1, \ldots, x_n) = -\sum_{i,k} J_{i,k} x_i x_k \ldots - J_{1,\ldots,n} \prod_s x_s \longleftrightarrow p(x_1, \ldots, x_n) = \frac{e^{-\beta \mathcal{H}(x_1,\ldots,x_n)}}{Z}$$





There are two basic types of high-order dependencies

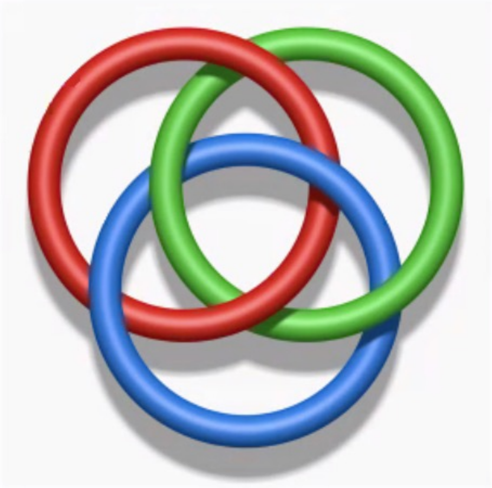




redundancy!

synergy!

- Synergy is the coexistence of **differentiation** (weak low-order relationships) and **integration** (strong collective properties).



Basis of consciousness in the brain (Tononi 2004)

In the 2D Ising model one observes nontrivial high order dependencies although mechanisms are pairwise

PHYSICAL REVIEW E **99**, 040101(R) (2019)

Rapid Communications

Synergy as a warning sign of transitions: The case of the two-dimensional Ising model

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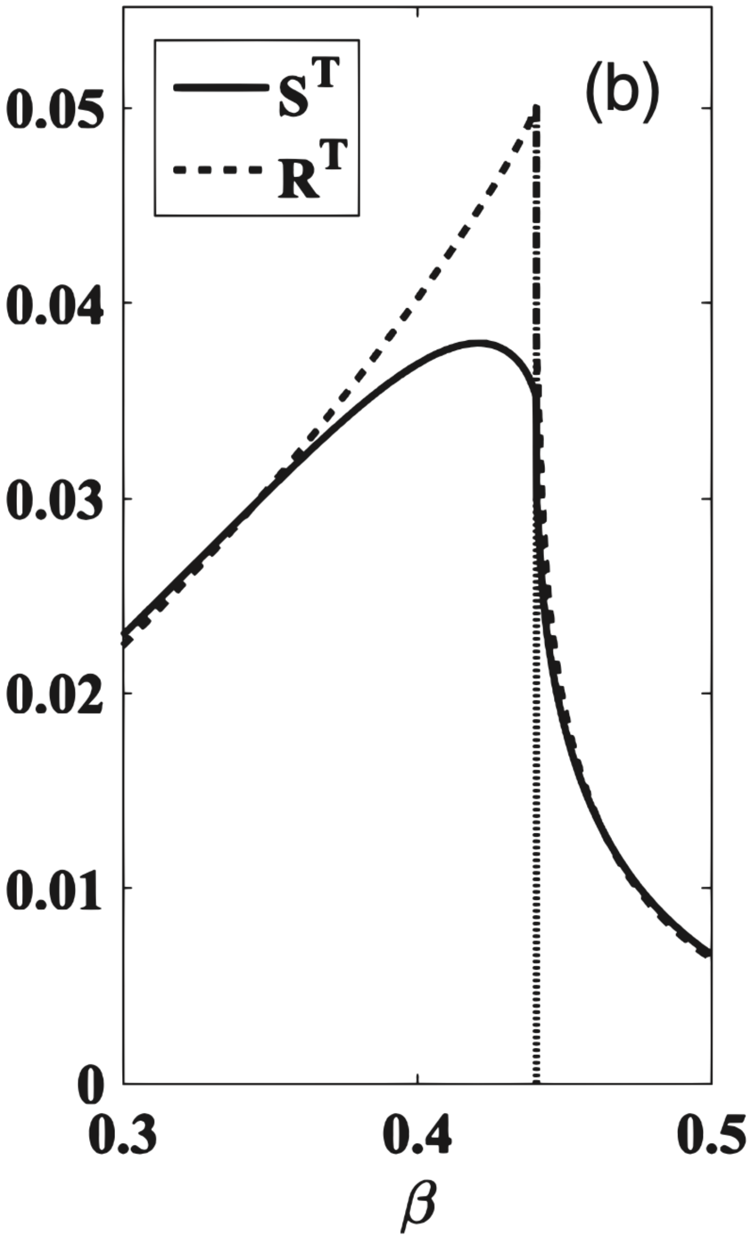

High order observables may arise from parwise interactions

PNAS
Proceedings of the National Academy of Sciences of the United States of America

Keyword, Author, or D

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RESEARCH ARTICLE

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Empirical social triad statistics can be explained with dyadic homophylic interactions

Tuan Minh Pham, Jan Korbel, Rudolf Hanel, and Stefan Thurner

+ See all authors and affiliations

PNAS February 8, 2022 119 (6) e2121103119; https://doi.org/10.1073/pnas.2121103119

Edited by Christopher Jarzynski, Institute for Physical Science and Technology, Department of Chemistry and Biochemistry and Department of Physics, University of Maryland at College Park, College Park, MD; received November 24, 2021; accepted December 27, 2021

Article | Figures & SI | Info & Metrics | PDF

Significance

Social stability is often associated with triangular interactions between people. Various possible social triangles appear in peculiar ratios. The triangles “The friend of my friend is my friend” and “The enemy of my friend is my enemy” are strongly overrepresented, which plays an important role for social balance. A standard explanation for these characteristic triangle fractions is that people consider triadic information before forming social relations. This assumption often contradicts everyday experience. We propose an explanation of the observed overrepresentations without individuals having to consider triangles. A society where individuals minimize their social stress self-organizes toward the empirically observed triangular structures. We demonstrate this with data from a society of computer game players, where triangle formation can be directly observed.

nature neuroscience

Article | Published: 26 May 2022

A synergistic core for human brain evolution and cognition

Andrea I. Luppi, Pedro A. M. Mediano, Fernando E. Rosas, Negin Holland, Tim D. Fryer, John T. O'Brien, James B. Rowe, David K. Menon, Daniel Bor & Emmanuel A. Stamatakis

Nature Neuroscience **25**, 771–782 (2022) | Cite this article

Mutual Information

Relative entropy between the joint distribution and the product distribution

$$I(X;Y) = D(p(x,y) \| p(x)p(y))$$

$$I(X;Y) = \sum_x \sum_y p(x,y) \log \frac{p(x,y)}{p(x)p(y)}$$

Reduction in the uncertainty of X due to the knowledge of Y

$$I(X;Y) = H(Y) - H(Y \mid X) =$$

$$H(X) + H(Y) - H(X,Y)$$

Transfer Entropy

X and Y two (vectorial) time series

x, the future values of X

If Y does not provide information about the future of x: Generalized Markov property

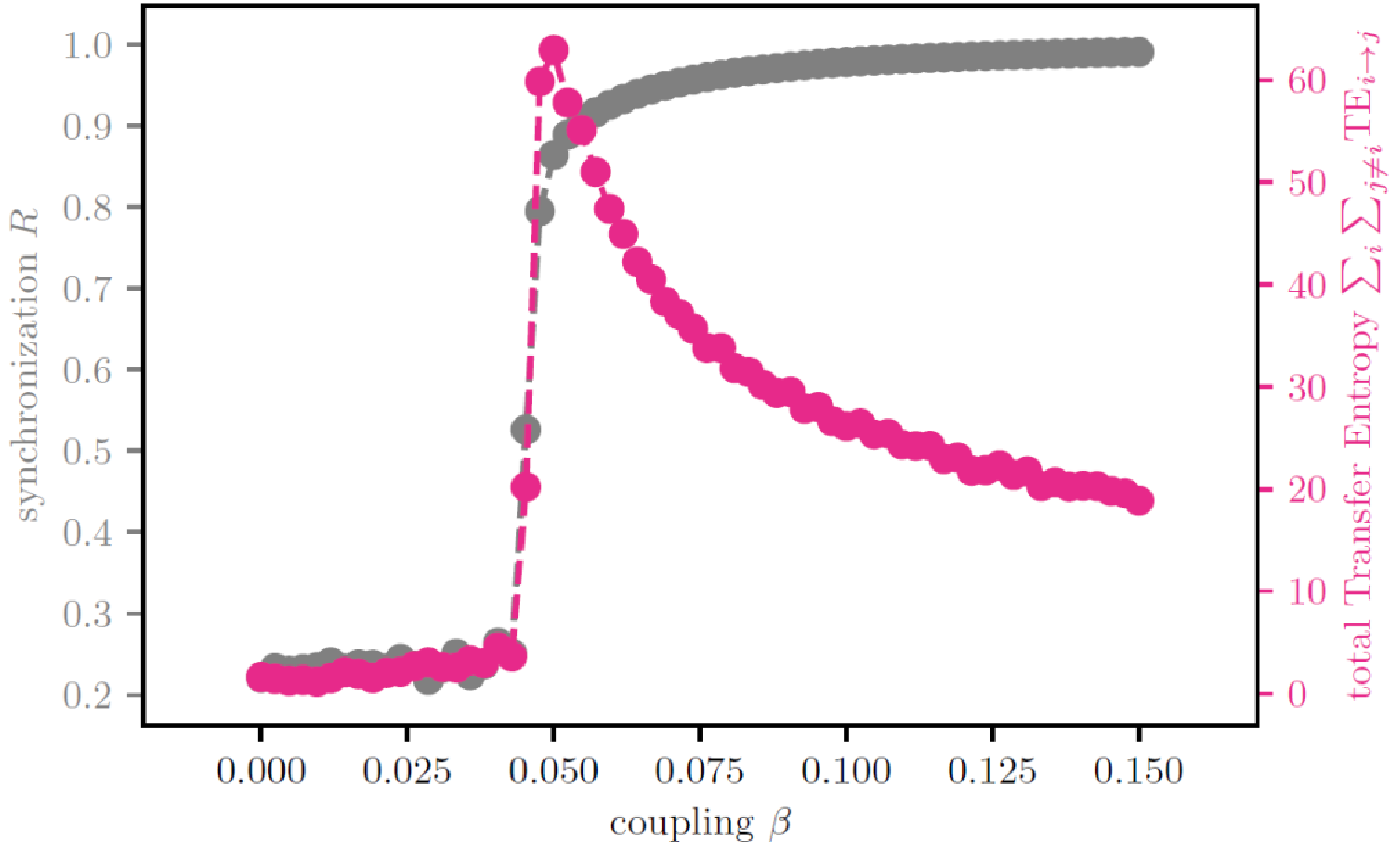
$$P(x \mid X) = P(x \mid X, Y)$$

$$T(Y \to X) = \int P(x, X, Y) \log\left(\frac{P(x \mid X, Y)}{P(x \mid X)}\right) dx\, dX\, dY$$

Transfer Entropy =I(x,Y|X)

INFORMATION DYNAMICS AT TRANSITIONS

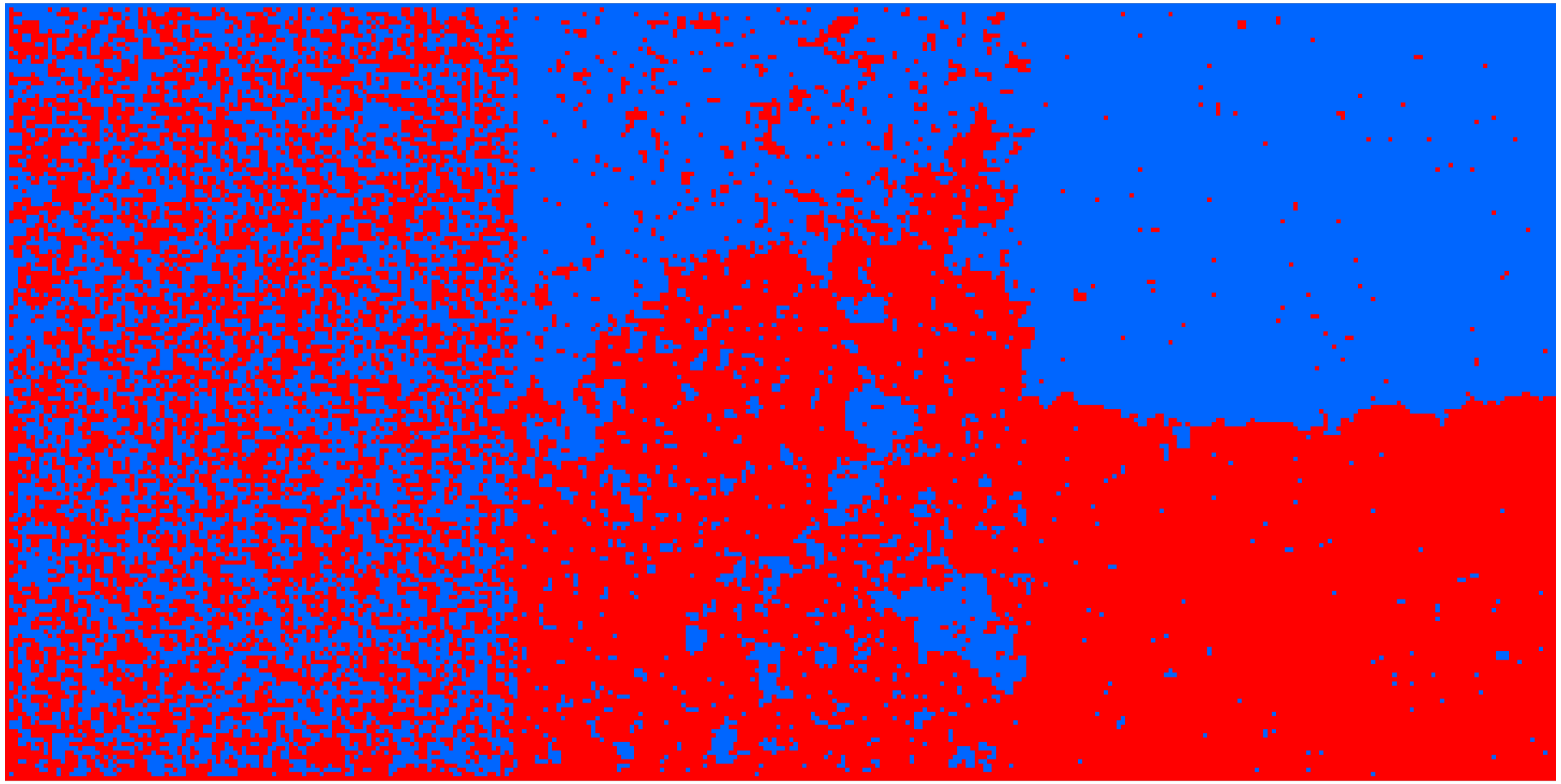
Information transfer is minimized both in the completely ordered and in the disordered state

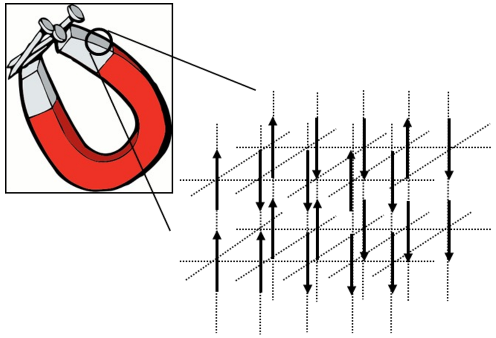


Kuramoto oscillators on a lattice (Heyvaert 2018)

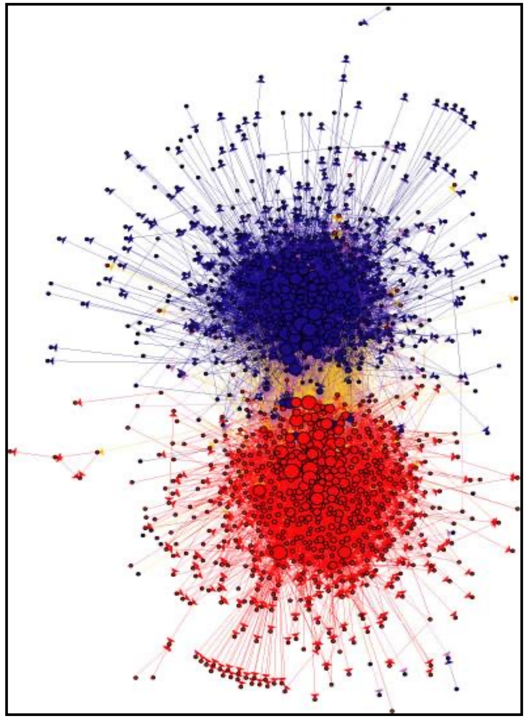
EXAMPLE OF TRANSITIONS : ISING MODEL

$T \to \infty$ $T \sim T_{crit}$ $T \to 0$



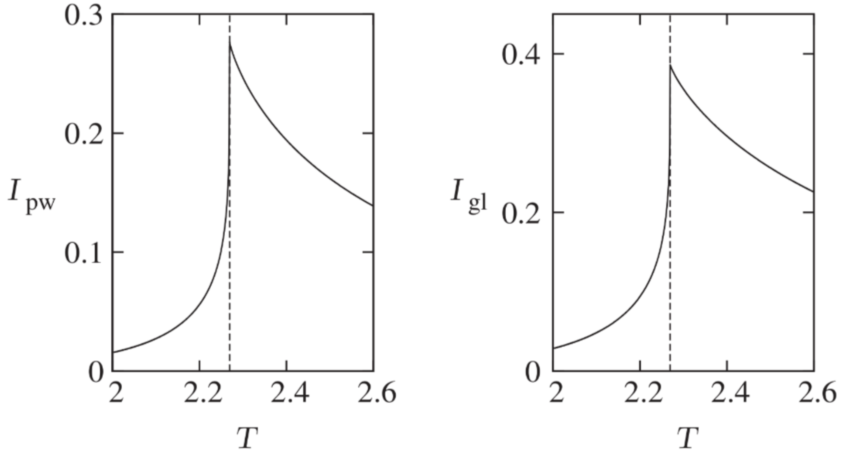


ISING MODEL: BEYOND FERROMAGNETISM



Polarization of news and opinions, financial crashes, epileptic seizures, learning, etc

INFORMATION DYNAMICS AT TRANSITIONS



Pairwise and global Mutual Information peak at the critical temperature (Matsuda et al. IJTP 1996).

Synergy as a warning sign of transitions

DECOMPOSITION FOR MUTUAL INFORMATION AND TRANSFER ENTROPY

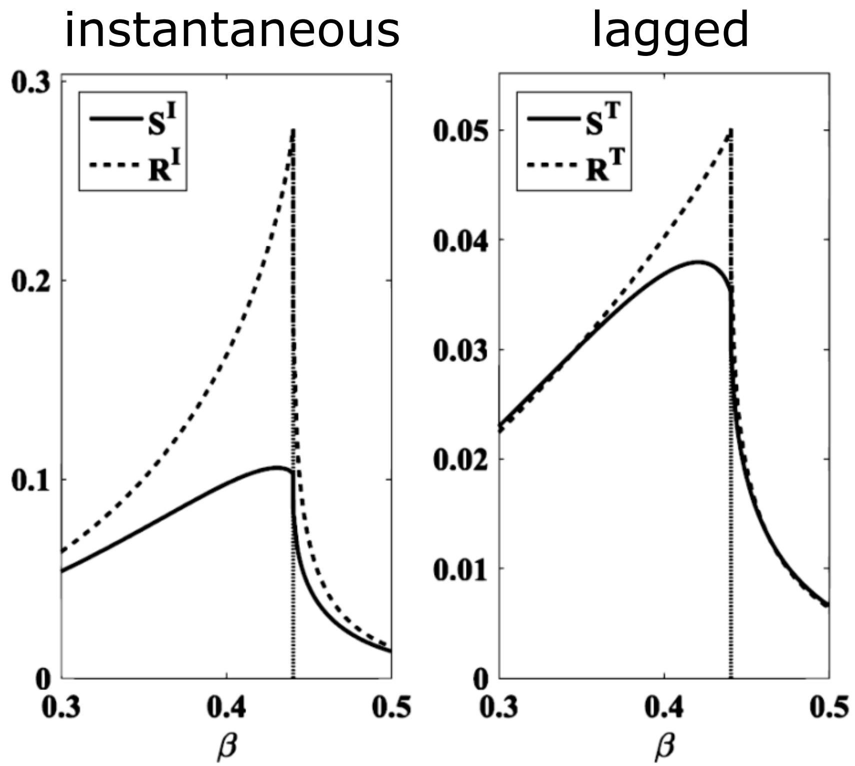
Not conditioning on the past (instantaneous)

$$
\begin{aligned}
I\left(s_i;\{s_j s_k\}\right) = U^I_{j\to i} + U^I_{k\to i} + R^I_{jk\to i} + S^I_{jk\to i},\\
I\left(s_i; s_j\right) = U^I_{j\to i} + R^I_{jk\to i},\\
I\left(s_i; s_k\right) = U^I_{k\to i} + R^I_{jk\to i}.
\end{aligned}
$$

Conditioning on the past (lagged)

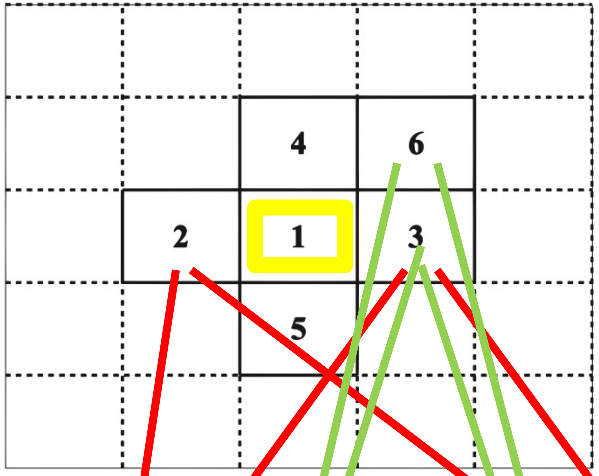
$$
\begin{aligned}
T_{jk\to i} = U^T_{j\to i} + U^T_{k\to i} + R^T_{jk\to i} + S^T_{jk\to i},\\
T_{j\to i} = U^T_{j\to i} + R^T_{jk\to i},\\
T_{k\to i} = U^T_{k\to i} + R^T_{jk\to i}.
\end{aligned}
$$

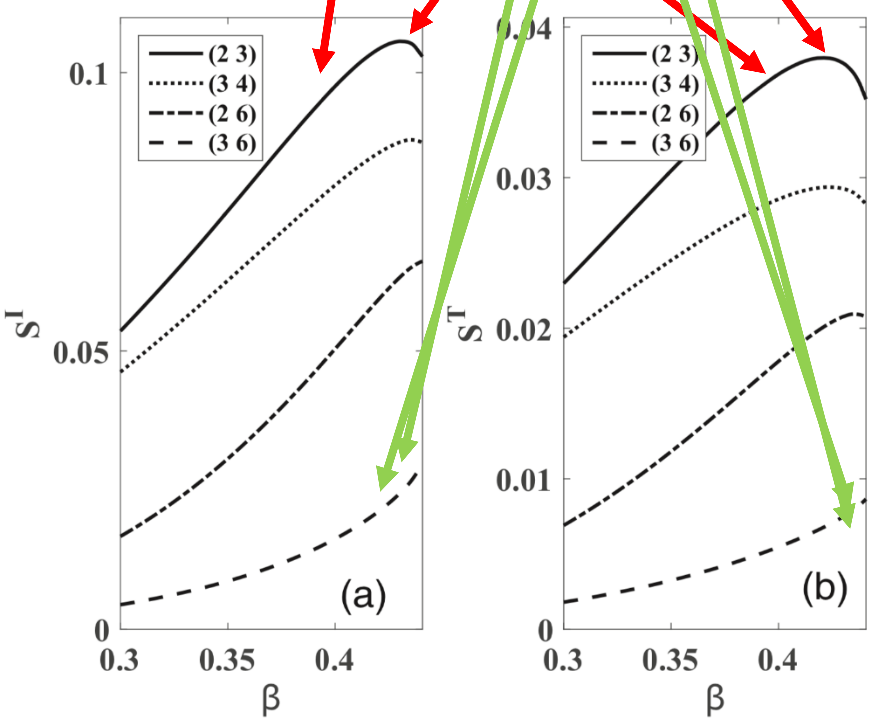
INFORMATION DYNAMICS AT TRANSITIONS



Is the synergy which peaks in the paramagnetic phase

The synergy peak approaches the critical value as the amount of synergy decreases





NOW ON THE HUMAN STRUCTURAL CONNECTOME

DTI of 196 subjects, age range 5-85 y

- does the synergy still peak before the critical point in a nonuniform network?
- are the hubs of structural connectivity also hubs of synergy?
- is there association with age?

NOW ON THE HUMAN STRUCTURAL CONNECTOME

Hubs of structural connectivity are not among the nodes towards which synergy is highest

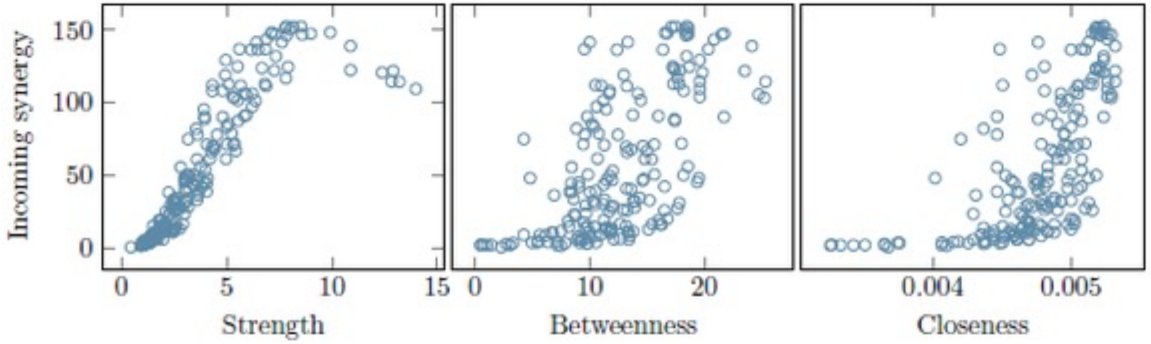


Fig 5. Comparing synergy with topological indices. From the left to the right, for each brain node the incoming synergy at criticality is compared with the strength of nodes, the betweenness and the closeness.

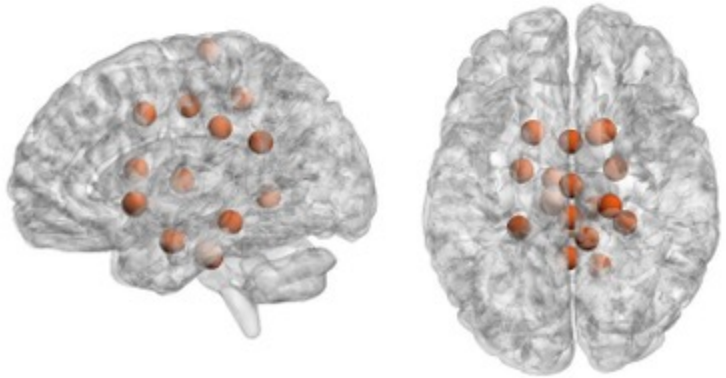
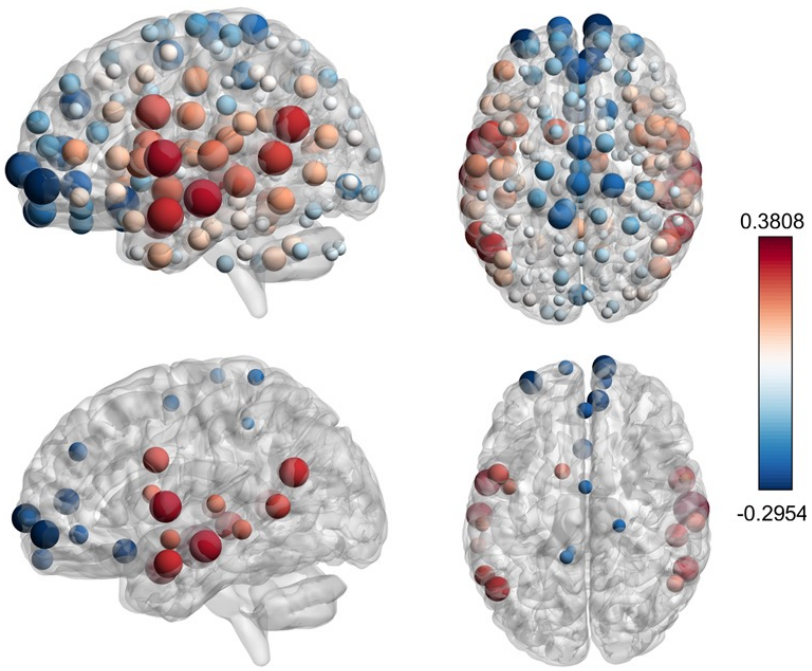


Fig 6. Hubs of synergy. Top nodes for the value of incoming synergy, radius and color of the spheres are arbitrary:
'Right Hippocampus', 'Brain Stem', 'Right Parahippocampal posterior' 'Left Parahippocampal posterior', 'Right Cingulate posterior', 'Right Precentral', 'Left Thalamus', 'Left Parahippocampal posterior', 'Left Hippocampus', 'Right Lingual', 'Right Caudate', 'Right Cingulate anterior'

NOW ON THE HUMAN STRUCTURAL CONNECTOME



Positive and negative associations of synergy with age, in localized clusters

In some regions this association is continuous with age, in other ones it's limited to the first ~30 years

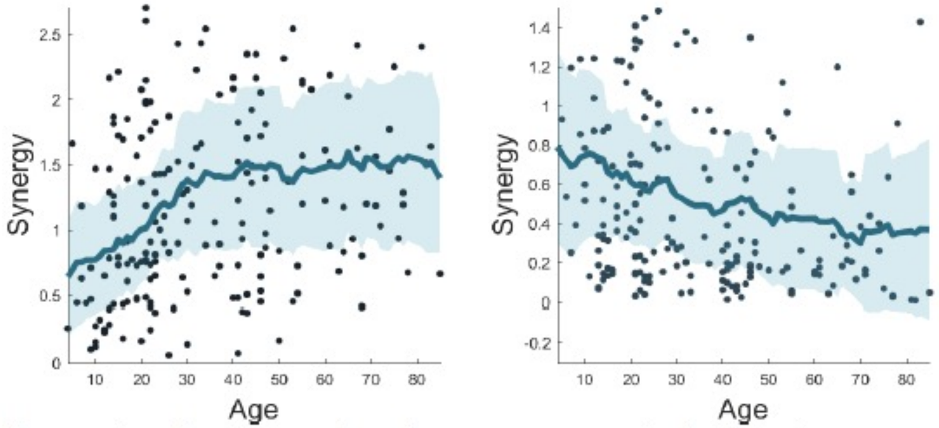
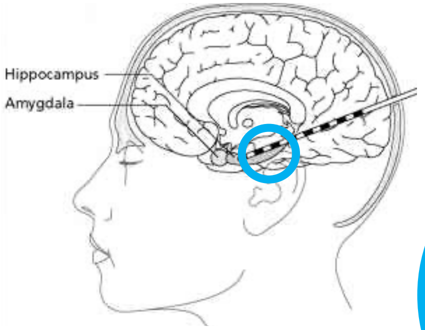
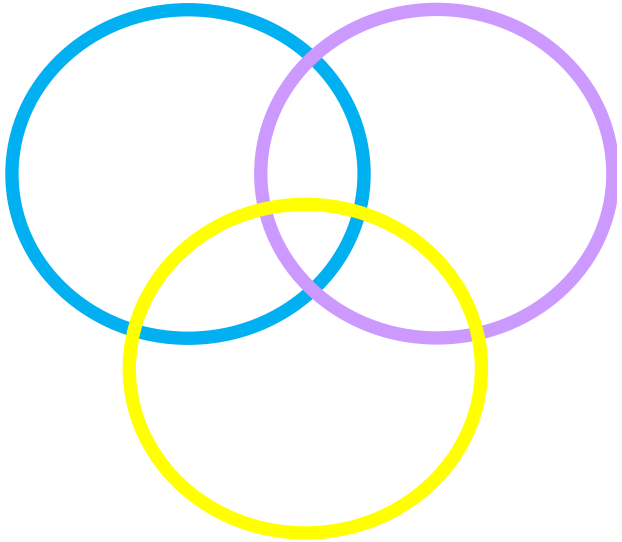
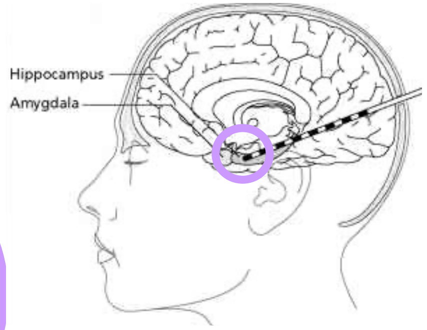


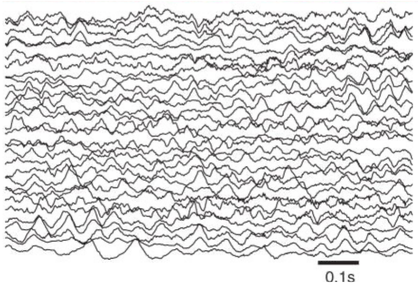
Fig 8. Scatter plot of synergy and age for two representative brain regions.
Left: Right Superior Temporal posterior, positive correlation.
Right: Right Frontal pole, negative correlation.
Local average and standard deviation are evaluated using the first 20 neighbours of each point.

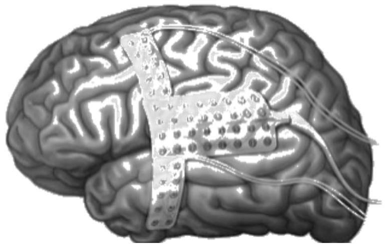
BRAIN SIGNALS: EPILEPSY







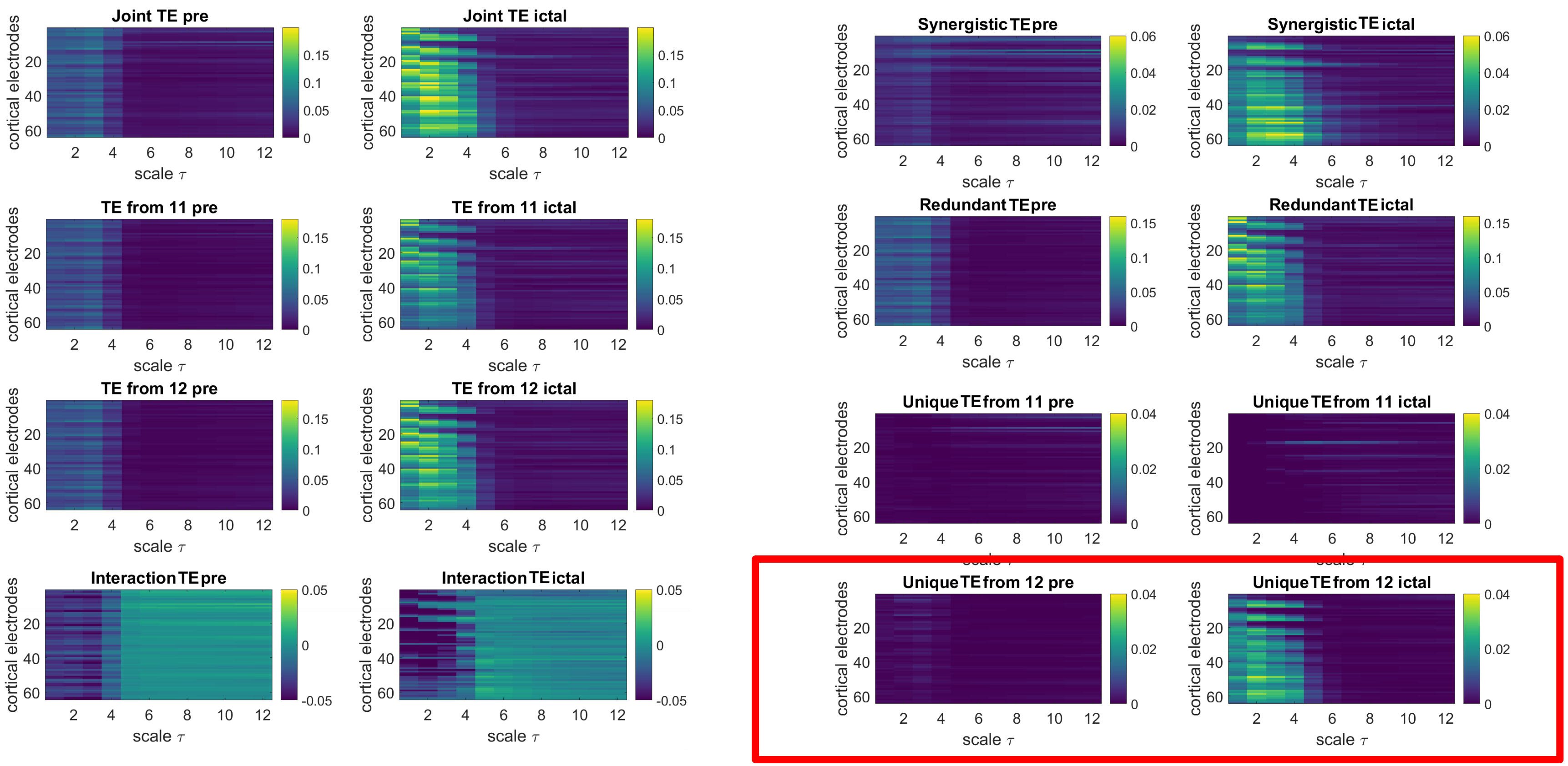




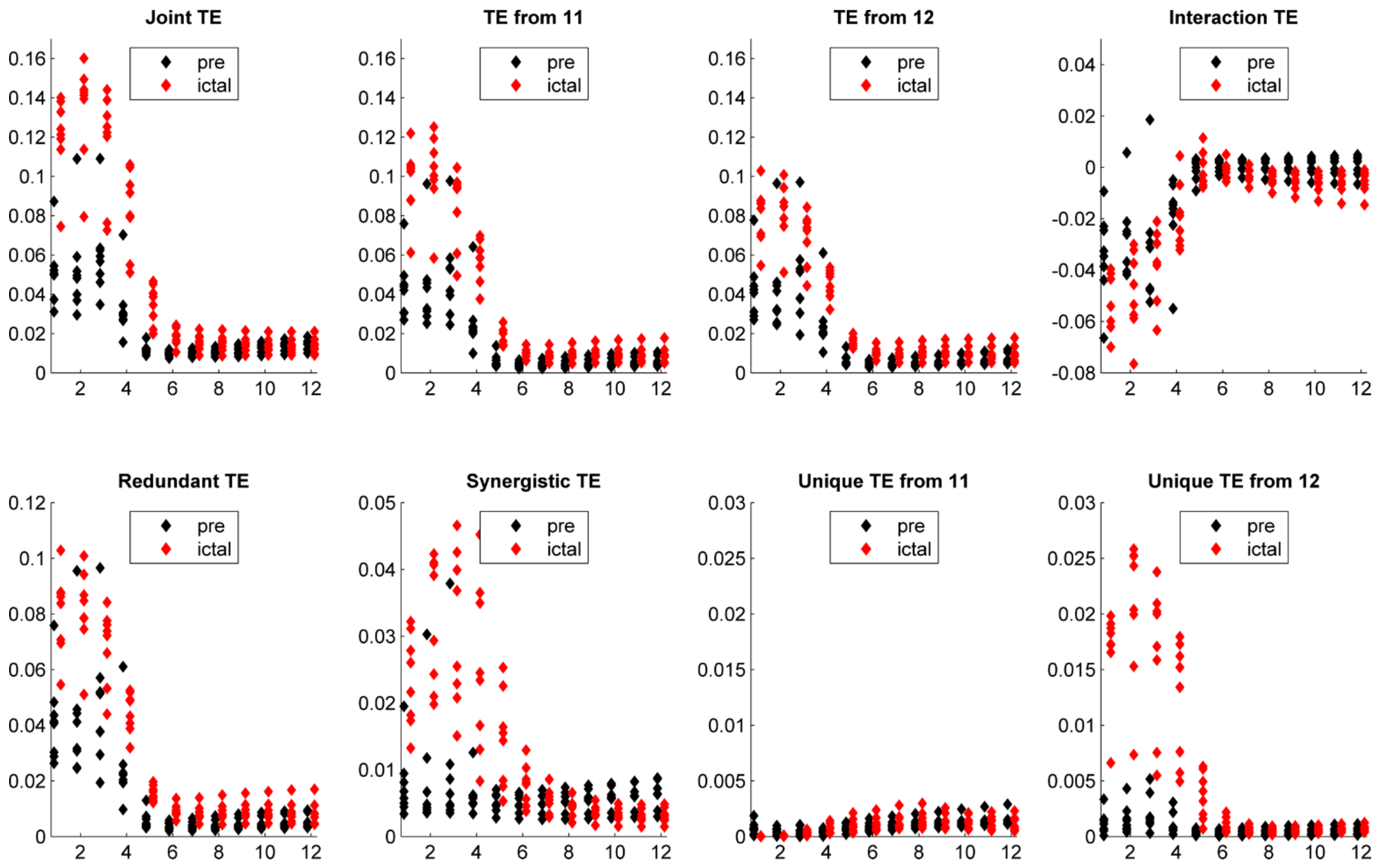
64 CORTICAL ELECTRODES AS TARGET, AND TWO DEPTH HIPPOCAMPAL ELECTRODES (11 AND 12, BOTH CANDIDATES AS EPILEPSY FOCI) AS DRIVERS

[M. Kramer et al., Epilepsy Research 79, 173-186, 2008]

BRAIN SIGNALS: EPILEPSY

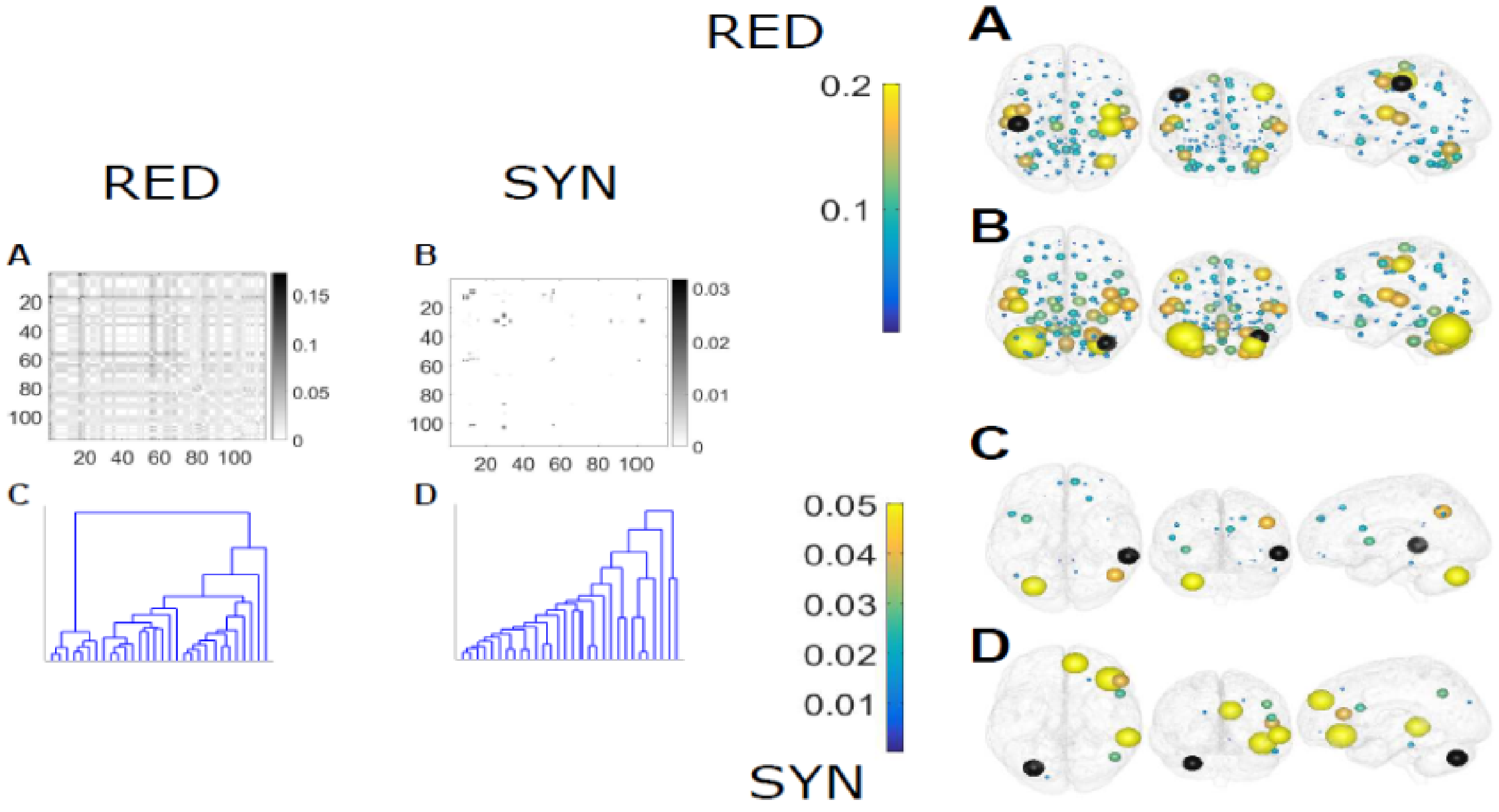


BRAIN SIGNALS: EPILEPSY



SYNERGY AND REDUNDANCY HAVE A HIERARCHICAL STRUCTURE

Regions forming redundant and synergetic multiplets with a representative region (black)



Hierarchical structure of synergy and redundancy networks

Resting state fMRI, Human Connectome Project

CHALLENGE:

- We have HOI mechanisms on one side, we have higher-order observables on the other side; what we sorely miss are inferential techniques to connect the two, and to be able to perform "higher-order mechanism selection" constrained by the observed behaviours over classes of higher-order mechanisms.

Thanks to:

- Tomas Scagliarini, Davide Nuzzi (Bari U)
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- Bryan Daniels (Arizona State U)