Synchronization and Chimera Phenomena in Neural Networks with with (alpha-stable Le´vy) Noise

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From: Barcelona, HongKong, Lodz, Oldenburg, Ponta Grossa, Potsdam, Sao Paulo, Xi‘an

Contents

- Introduction
- (Old) Chimeras in two coupled oscillator ensembles (2004)
- Chimera-like states in a neural network model of the cat brain
- Coherence-resonance chimeras (2022)
- Conclusions

Network Physiology

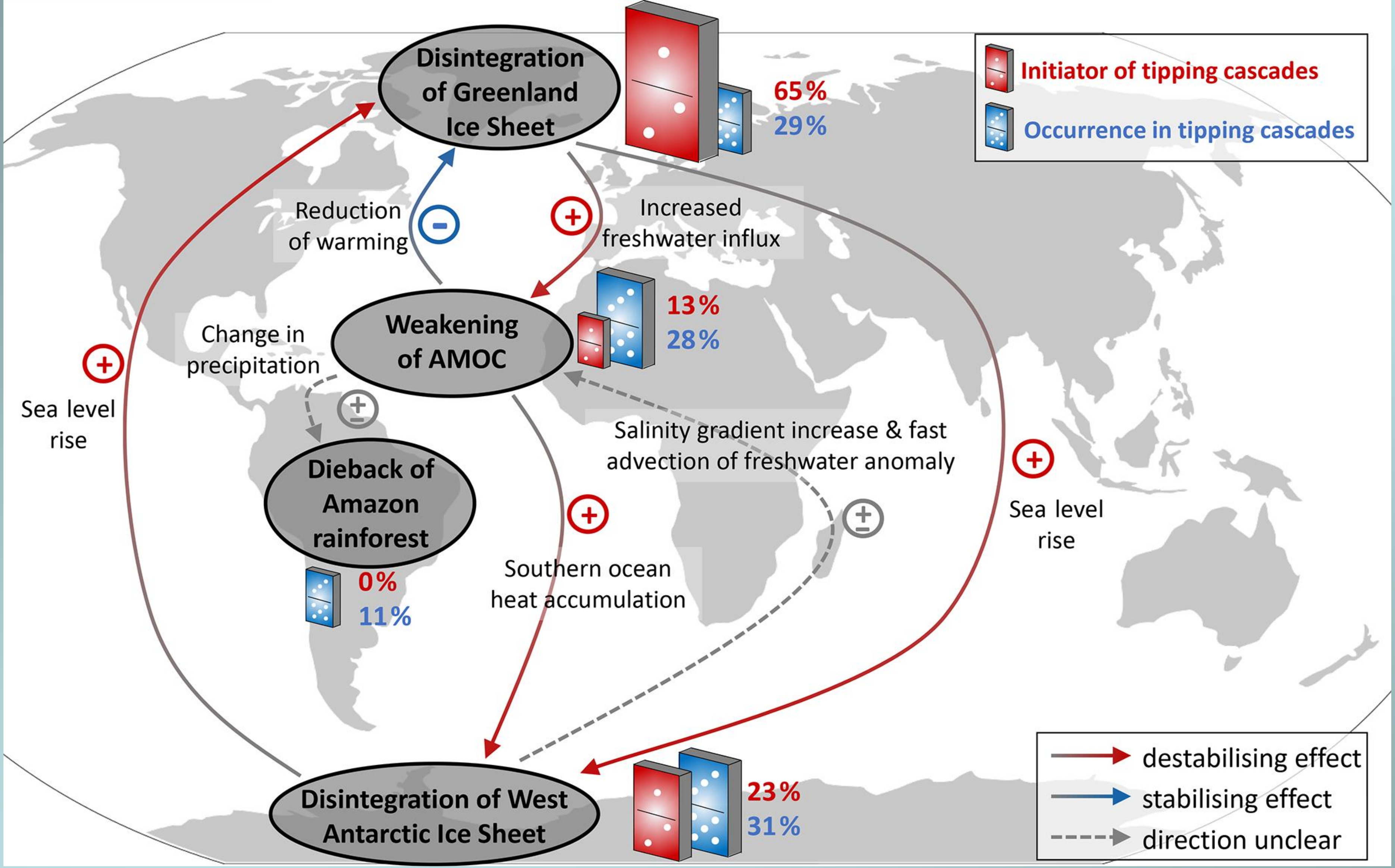
Plamen – created this promising approach

some similarities in System Earth

Challenge:

modeling as complex network – **interaction** of tipping areas

Cascading (domino-like) or isolated?



Interacting tipping elements increase risk of climate domino effects under global warming

Earth Syst. Dynam., 12, 601–619, 2021

Complex Networks:

Formation of **Heterogeneity**

(in **dynamic** behaviour)

Synchronization
(phase, amplitude)

Full - Partial

Clustering

Chimera…

Chimera

- **Chimera** states are hybrid states characterized by the coexistence of localized synchronized and unsynchronized dynamics (partial synchro)
- Non-trivial phenomena occurring due to symmetry breaking
- First documented:
  Kuramoto/Battogtokh (2002) (791 cit)
  name given by Abrams/Strogatz (2004) (914 cit)
- Now popular topic in complex systems science and applications

Our first „Chimera“ contribution

PHYSICAL REVIEW E **70**, 056125 (2004)

Synchronization of two interacting populations of oscillators

Ernest Montbrió, Jürgen Kurths, and Bernd Blasius
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(Received 24 March 2004; published 22 November 2004)

We analyze synchronization between two interacting populations of different phase oscillators. For the important case of asymmetric coupling functions, we find a much richer dynamical behavior compared to that of symmetrically coupled populations of identical oscillators. It includes three types of bistabilities, higher order entrainment, and the existence of states with unusual stability properties. All possible routes to synchronization of the populations are presented and some stability boundaries are obtained analytically. The impact of these findings for neuroscience is discussed.

But unfortunately not using the fancy term CHIMERA (cit 152)

Another Paper in 2008

PRL **101,** 084103 (2008) PHYSICAL REVIEW LETTERS week ending 22 AUGUST 2008

Solvable Model for Chimera States of Coupled Oscillators

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Networks of identical, symmetrically coupled oscillators can spontaneously split into synchronized and desynchronized subpopulations. Such chimera states were discovered in 2002, but are not well understood theoretically. Here we obtain the first exact results about the stability, dynamics, and bifurcations of chimera states by analyzing a minimal model consisting of two interacting populations of oscillators. Along with a completely synchronous state, the system displays stable chimeras, breathing chimeras, and saddle-node, Hopf, and homoclinic bifurcations of chimeras.

Two population model

$$\dot{\theta}_i^{(1,2)} = \omega_i^{(1,2)} - \frac{K_p}{N}\sum_{j=1}^{N} \sin(\theta_i^{(1,2)} - \theta_j^{(1,2)} + \alpha)$$

$$- \frac{K}{N}\sum_{j=1}^{N} \sin(\theta_i^{(1,2)} - \theta_j^{(2,1)} + \alpha),$$

K_p - internal coupling

K – interpopulation coupling

α - phase shift – asymmetry in coupling

Order Parameter

$$R^{(1,2)}e^{i\psi^{(1,2)}} = (1/N)\Sigma_{j=1}^{N} e^{i\theta_j^{(1,2)}}$$

Order parameter

$$\dot{\theta}_i^{(1,2)} = \omega_i^{(1,2)} - K_p R^{(1,2)} \sin(\theta_i^{(1,2)} - \psi^{(1,2)} + \alpha) - K\, R^{(2,1)} \sin(\theta_i^{(1,2)} - \psi^{(2,1)} + \alpha).$$

Model via order parameter

$g^{(1,2)}(\omega)$ Frequency distribution of width γ

Bifurcation parameters: K, α and γ

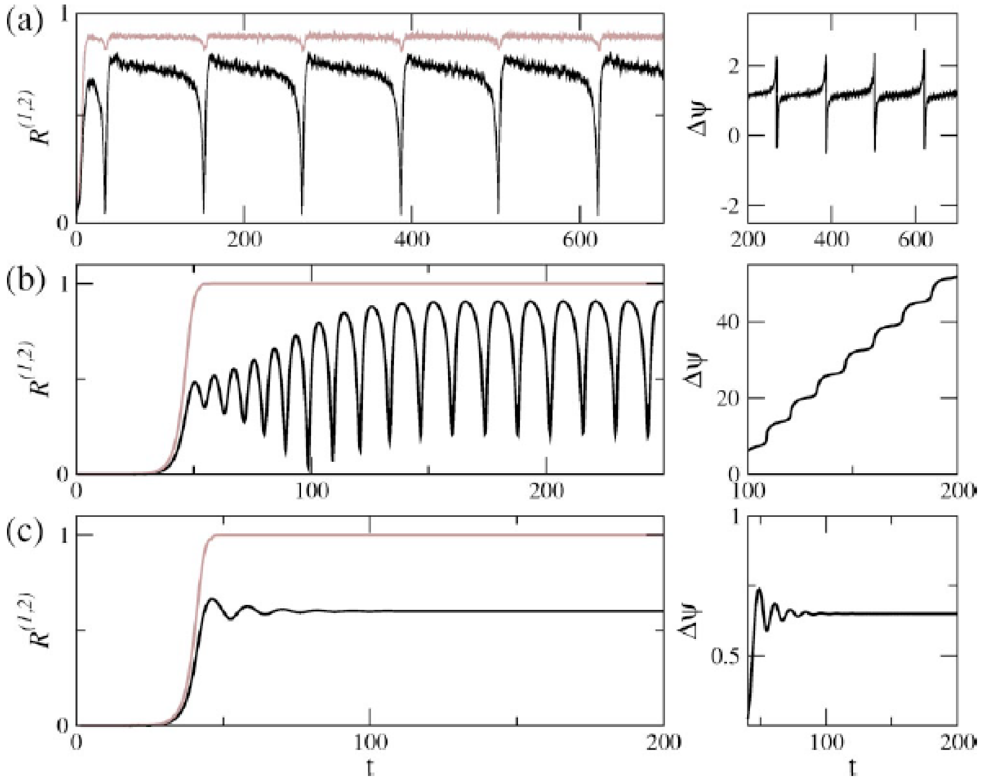


FIG. 3. Order parameters $R^{(1)}$ (black) and $R^{(2)}$ (gray) and phase difference $\Delta\psi$ as a function of time ($N=1000, \Delta\omega=0.5, \overline{\omega}=0, K_p=1$). At $t=0$ the phases were equally spaced in $(0, 2\pi]$. (a) $\alpha=\pi/4, \gamma=0.12, K=0.53$ (region II) [Fig. 2(h)]; (b) and (c) represent regions II′($\alpha=1.2, K=0.8$) and I′($\alpha=1.2, K=1.05$), respectively ($\gamma=0$, see Fig. 4).

b) State 1 (we) –

Breathing chimera (Abrams)

c) State 2 (we) –

Stable chimera (Abrams)

Complex Networks

Example:
Brain Dynamics

Concept: **Network of Networks**

(Anatomy vs. Functionality)

Network from measured anatomic connectivity

System Brain: Cat Cerebal Cortex

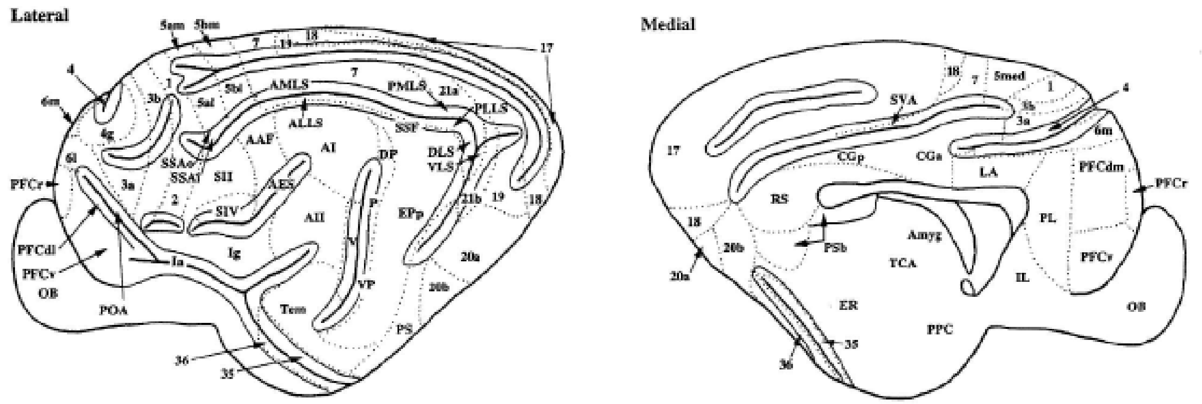


Fig. 1. Topographical map of cat cerebral cortex (from [20]).

Connectivity

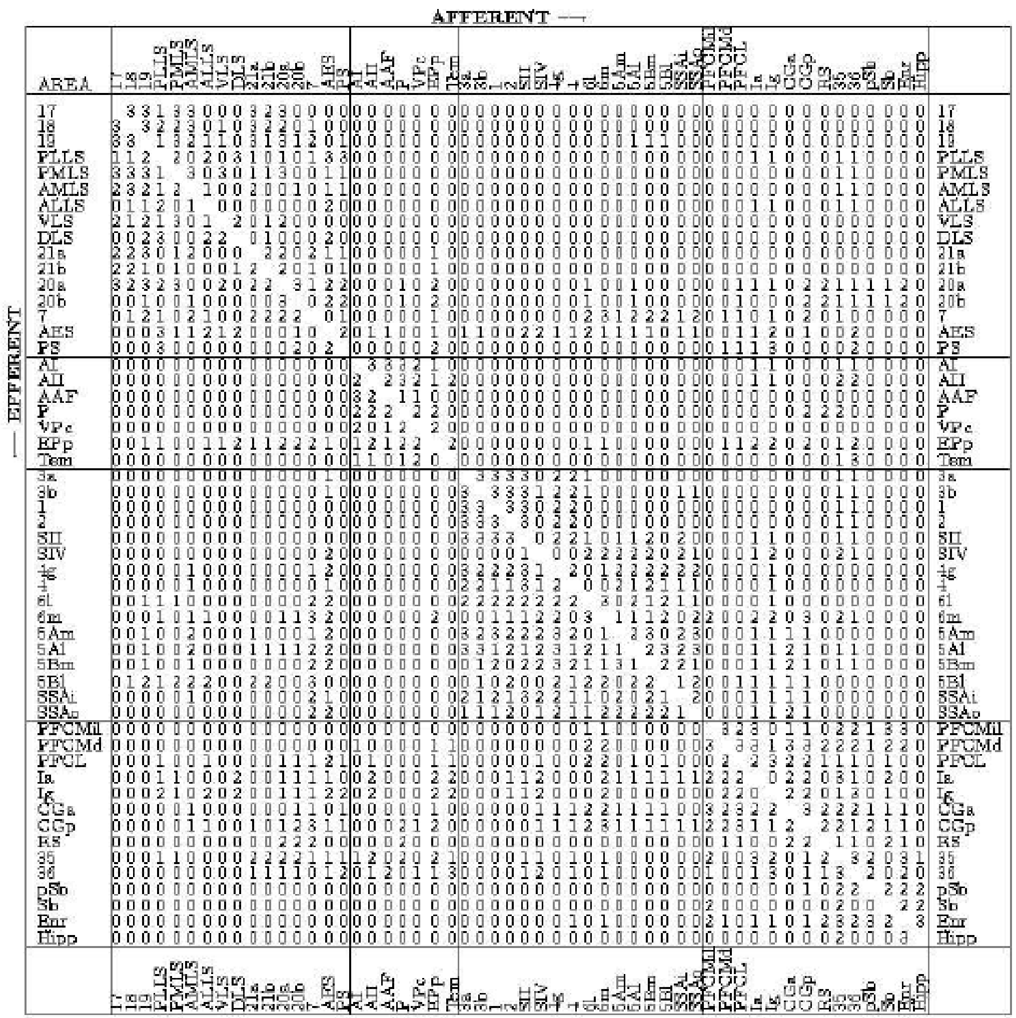


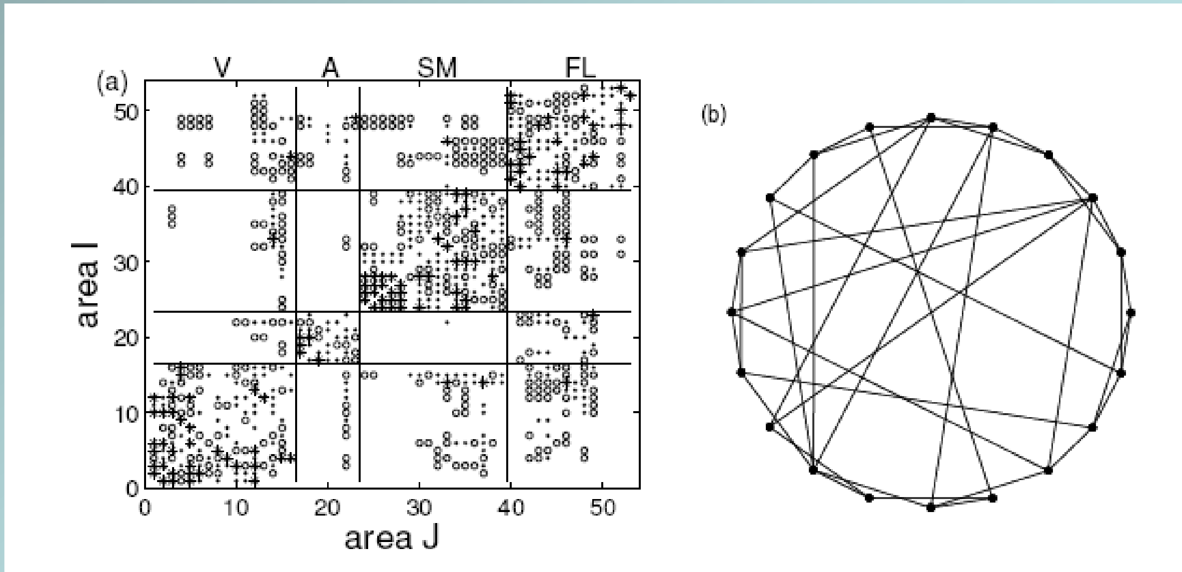
Fig. 2. Connectivity matrix representing connections between 53 cortical areas of cat brain.

Scannell et al.,

Cereb. Cort., 1999

Analysis of the network structure

Hierarchical organization in complex brain networks

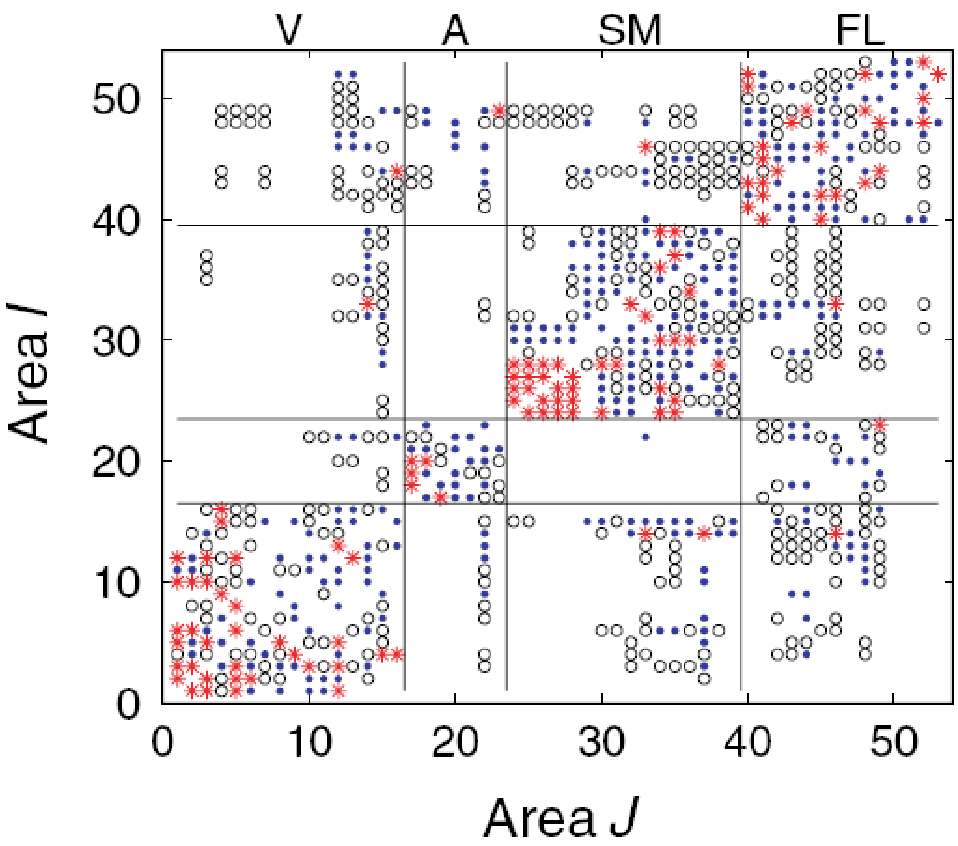


a) Connection matrix of the cortical network of the cat brain (anatomical)

b) Small world sub-network to model each node in the network (200 nodes each, FitzHugh Nagumo neuron models - excitable)

➔ **Network of networks**

Phys Rev Lett 97 (2006), Physica D 224 (2006), New J Phys (2007), CHAOS (2008), CHAOS (2009), Frontiers of Neuroscience (2010)

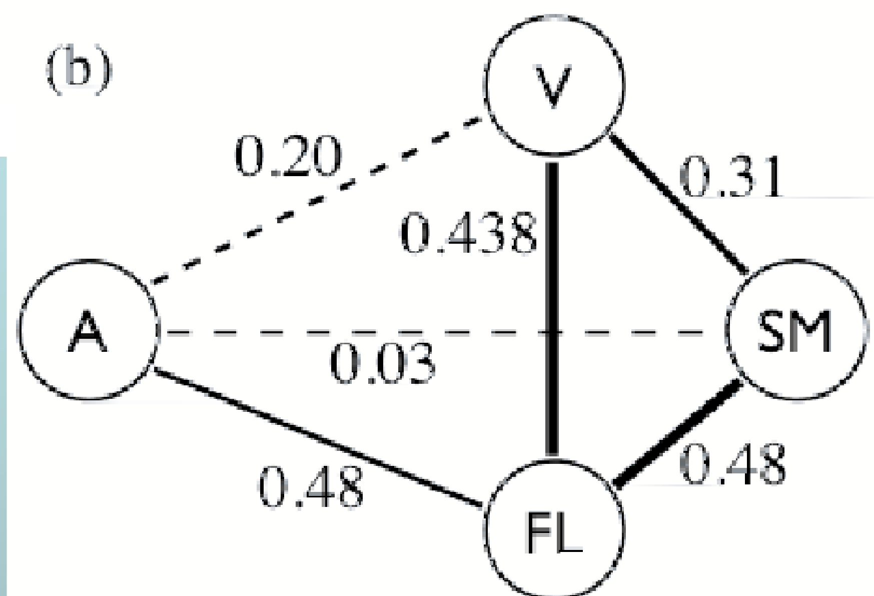


Density of connections between the four communities

•Connections among the nodes: 2 … 35

•830 connections

•Mean degree: 15



Parameter: Betweenness

Betweenness Centrality B

Number of shortest paths that connect nodes j and k n_{jk}

Number of shortest paths that connect nodes j and k AND path through node i $n_{jk}(i)$

Local betweenness of node i

$$b_i = \sum_{j,k,j \neq k} \frac{n_{jk}(i)}{n_{jk}}$$

(local and global aspects included!)

Betweenness Centrality B = < b_i >

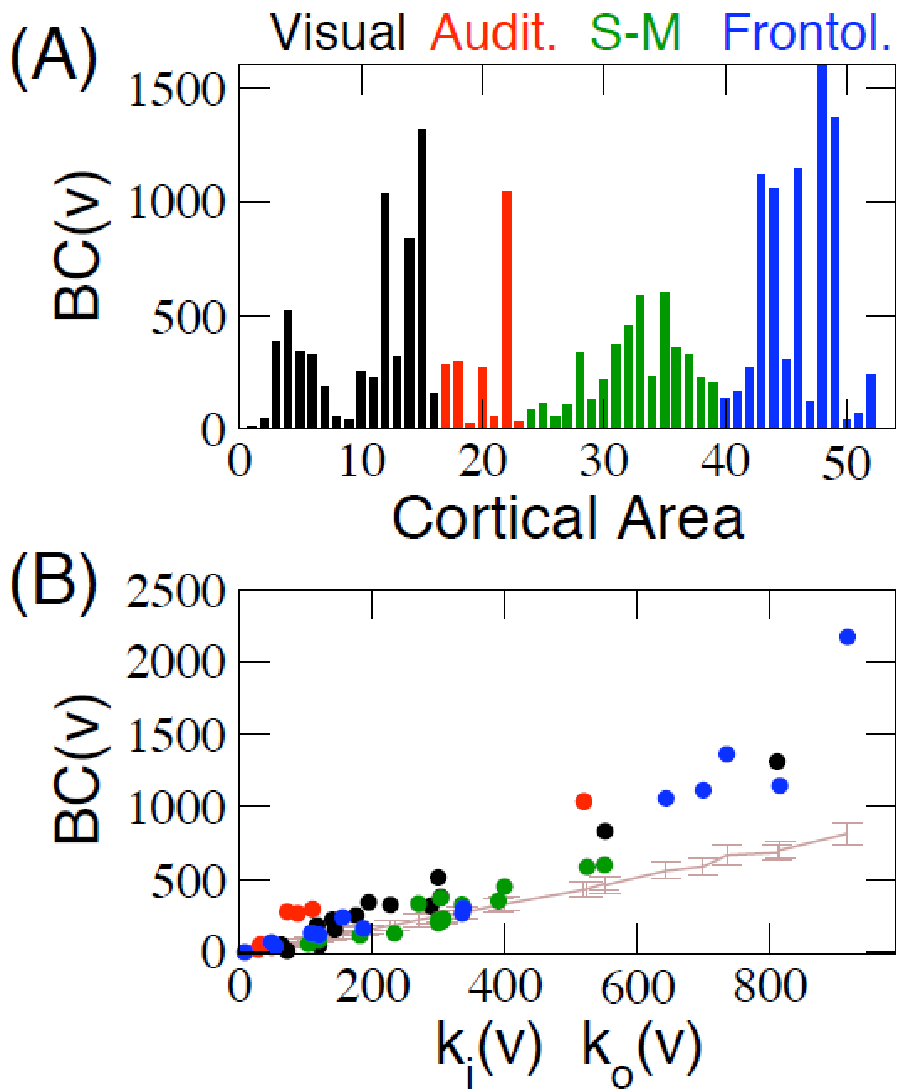


FIG. 4: Centrality of cortical areas. (**A**) Betweenness of cortical areas shows that at each sensory system few areas are very central. (**B**) Comparison between BC of cortical areas and the expected centrality due to their degree (brown line). As a consequence of the modular and hierarchical organisation of the network, low degree areas closely follow the expected centrality, but hubs are significantly more central than expected. Communication paths between sensory systems are centralised through the hubs.

Major features of organization of cortical connectivity

- Large density of connections (many direct connections or very short paths – fast processing)
- Clustered organization into functional com-munities
- Highly connected hubs (integration of multisensory information) - **superhubs**

Data-based network construction

➔ Modelling

Combination:
macroscopic (data based) &

mesoscopic (FHN neuron models 10000 elements)

Model for neuron i in area I

$$\epsilon \dot{x}_{I,i} = f(x_{I,i}) + \frac{g_1}{k_a}\sum_{j}^{N_a} M_I^L(i,j)(x_{I,j} - x_{I,i})$$
$$+ \frac{g_2}{\langle w \rangle}\sum_{J}^{N} M^C(I,J)L_{I,J}(i)(\bar{x}_J - x_{I,i}),$$
$$\dot{y}_{I,i} = x_{I,i} + a_{I,i} + D\xi_{I,i}(t),$$

where

$$f(x_{I,i}) = x_{I,i} - \frac{x_{I,i}^3}{3} - y_{I,i}.$$

FitzHugh Nagumo model

D = 0.03

$a \in (1.05, 1.15)$

$\epsilon = 0.01$

SW network p = 0.3

Transition to synchronized firing

g – coupling strength – control parameter

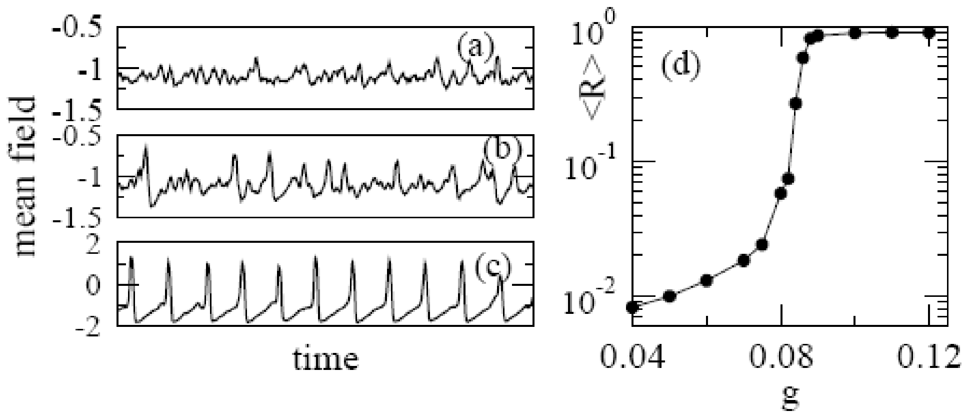


FIG. 2: Mean activity X of one area at various couplings (a) $g = 0.06$, (b) $g = 0.082$ (c) $g = 0.09$. The average correlation coefficient $\langle R \rangle = \frac{1}{N(N-1)} \sum_{I \neq J}^{N} R(I, J)$ $(N = 53)$ vs. g.

Possible interpretation: functioning of the brain near a 2nd order phase transition

Functional Organization vs. Structural (anatomical) Coupling

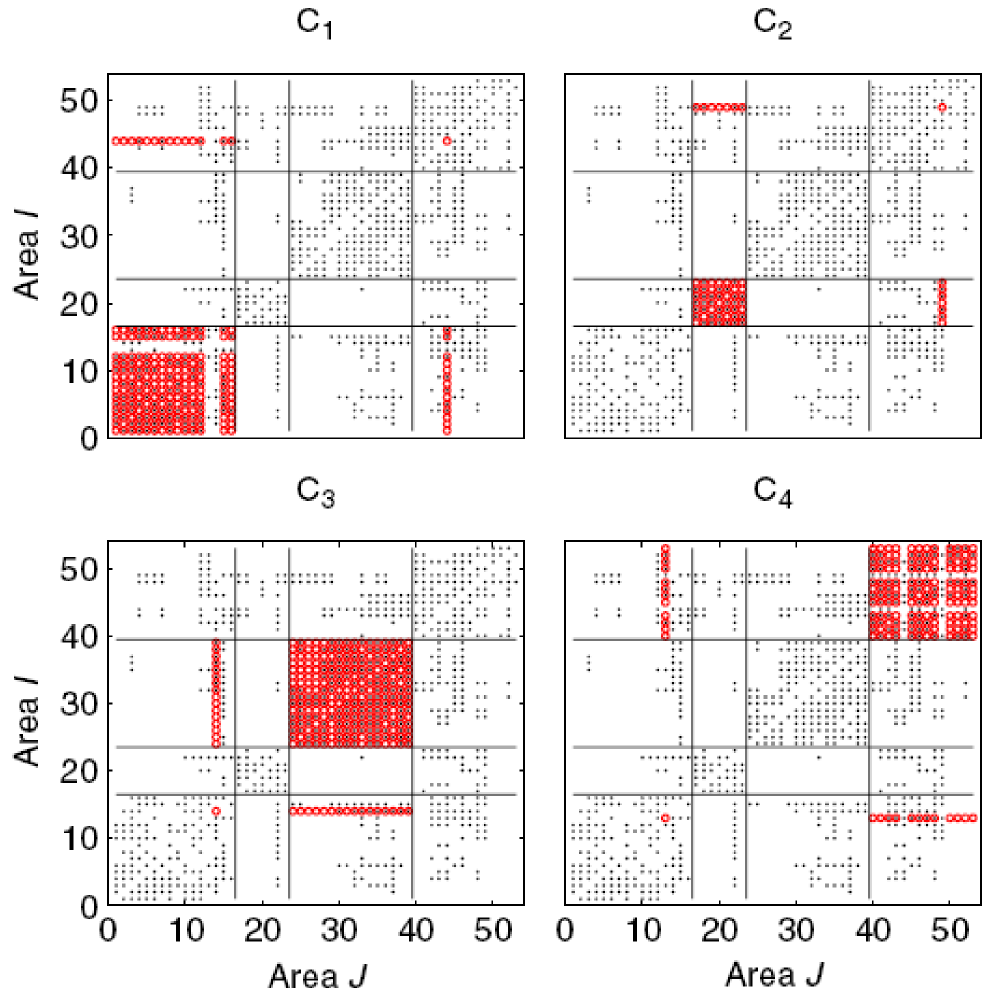


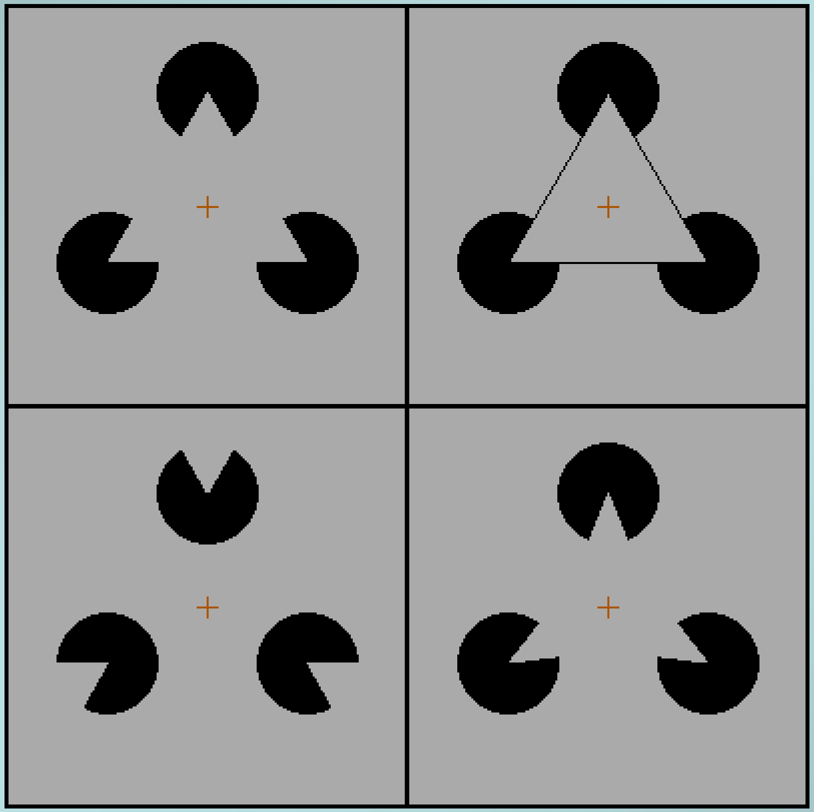
Figure 14. Four major dynamical clusters (○) with weak coupling strength $g = 0.07$, compared to the underlying anatomical connections (·).

Cognitive Processes

Problem: Understanding processing of visual stimuli

- Kanizsa-figure as stimulus (virtual figure vs. control figure)
- EEG-measurements (500 Hz, 30 channels)
- Multivariate synchronization analysis to identify **synchronized clusters**

Kanizsa Figures



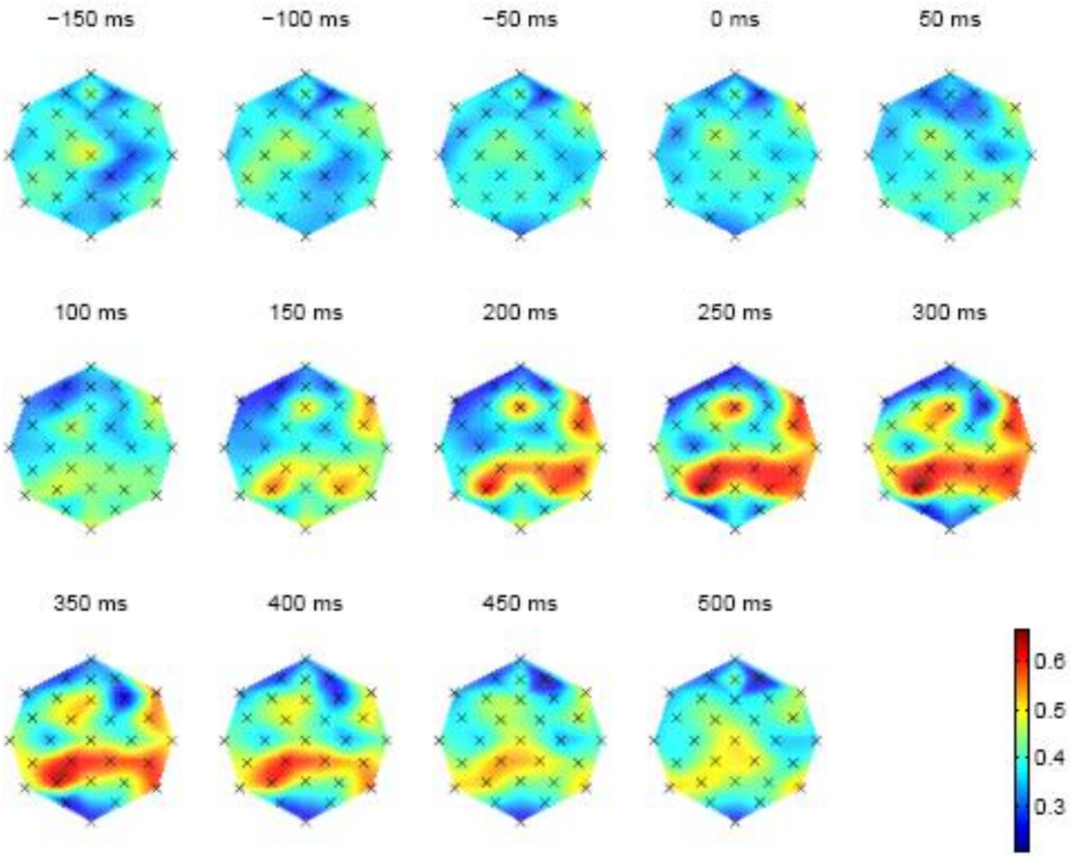
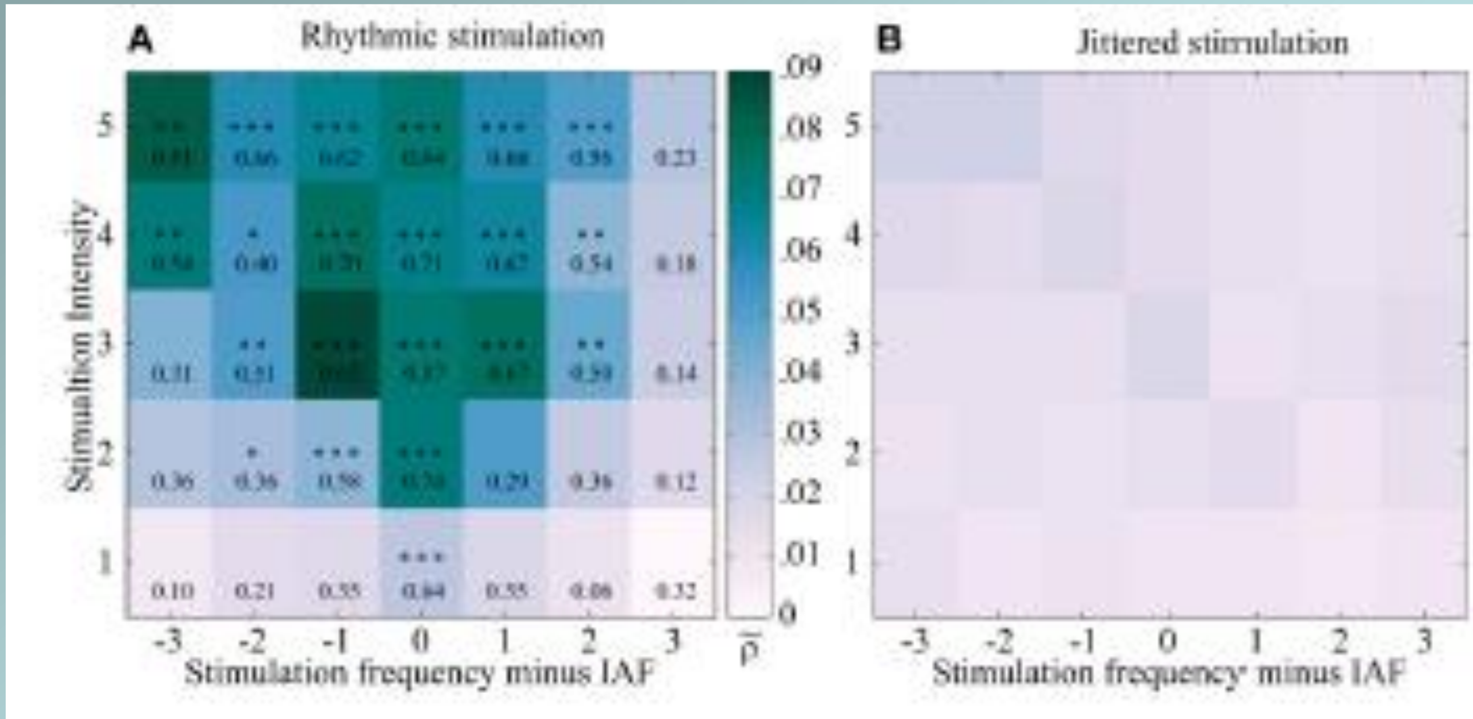


Figure 3: Time evolution of the cluster synchronization topography at $f = 13$ Hz for the Kanizsa condition. The continuous colors correspond to an interpolation of the ρ_{iC}-values attributed to the electrodes, whose positions are marked by $\times$-symbols. For a chart of the electrode names, see Fig. 5.

Brain synchronization via light stimulation

- Light stimulation via LED flickerng at 5 different light intensities and different frequencies in **rhythmic** and **jittered** regimes
- EEG measurements – Pz used for analysis
- Sampling 1000 Hz
- 5 x 7 intensity-frequency combinations
- Intrinsic frequency in alpha-range determined
- Phases via Hilbert trafo
- **Problem:** modify brain oscillations?



- Significant Arnold-like tongue for rhythmic stimulation

- No indication for jittered stimulation

- Offers rather simple ways to change brain oscilations

Notbohm, Kurths, Herrmann, Front. Human Neurosc. 2016

2. Macroscopic Modelling (2017)

Now also looking for Chimera-like states

Network model

- **Topology:** Cat brain (as before but a bit more refined)
- 65 nodes and 1139 connections
- **Dynamics:** Hindmarsh-Rose neuronal model

HR model

$$\dot{x}_j = y_j - x_j^3 + bx_j^2 + I_j - z_j - \frac{\alpha}{n'_j}\sum_{k=1}^{N} G'_{j,k}\Theta(x_k)$$
$$-\frac{\beta}{n''_j}\sum_{k=1}^{N} G''_{j,k}\Theta(x_k),$$
$$\dot{y}_j = 1 - 5x_j^2 - y_j,$$
$$\dot{z}_j = \mu\left[s\left(x_j - x_{\text{rest}}\right) - z_j\right],$$

G – coupling matrix (0, 1/3, 2/3, 1)
I – membrane input current (= 4.4)
α - intra connection strength (inside an area/community)
β - inter „ (between areas/communities)

$$\Theta(x_k) = (x_j - x_{\text{rev}})[1 + e^{-\lambda(x_k - \theta)}]^{-1}$$

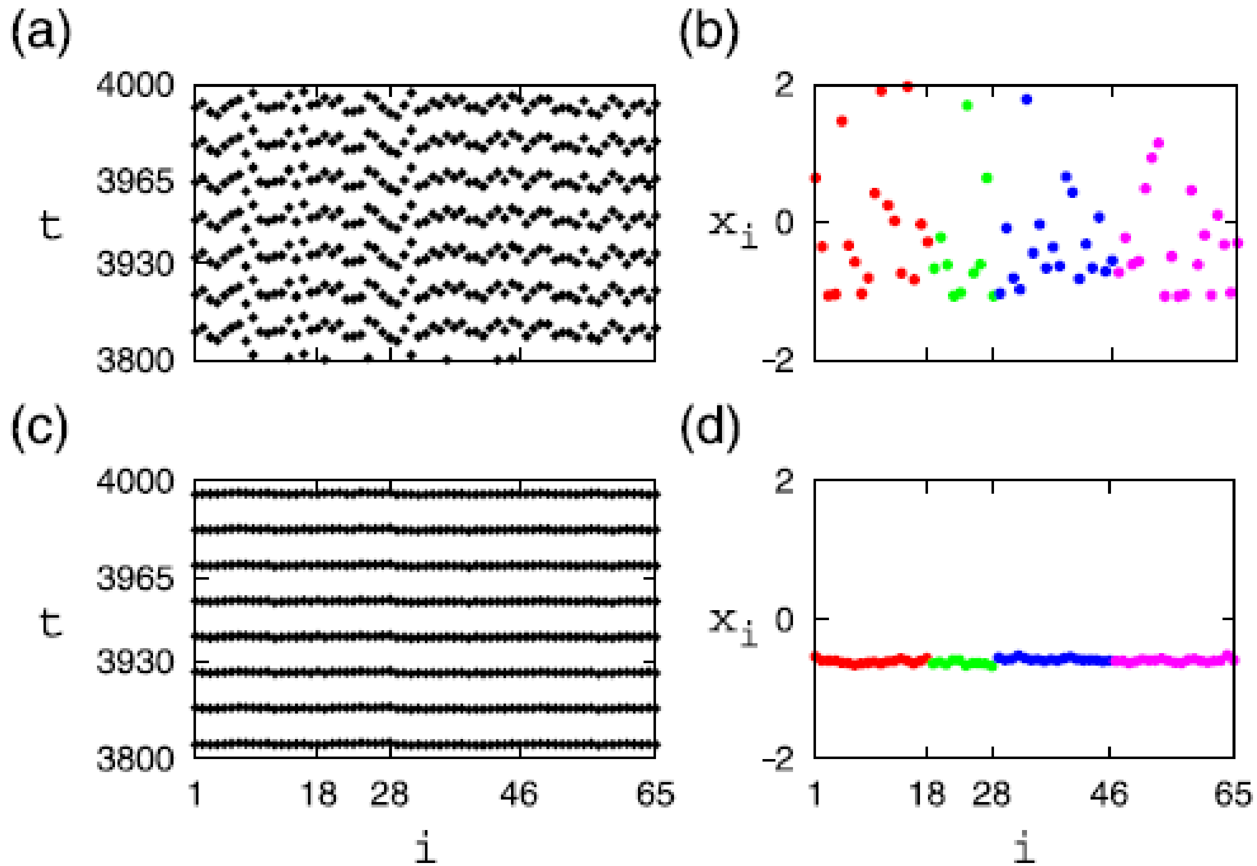


Fig. 2. Space-time plots (left) and snapshot of the variable x (right). (a) and (b) exhibit desynchronous behaviour for $\alpha = 0.001$ and $\beta = 0.001$. (c) and (d) show synchronous behaviour for $\alpha = 0.21$ and $\beta = 0.04$.

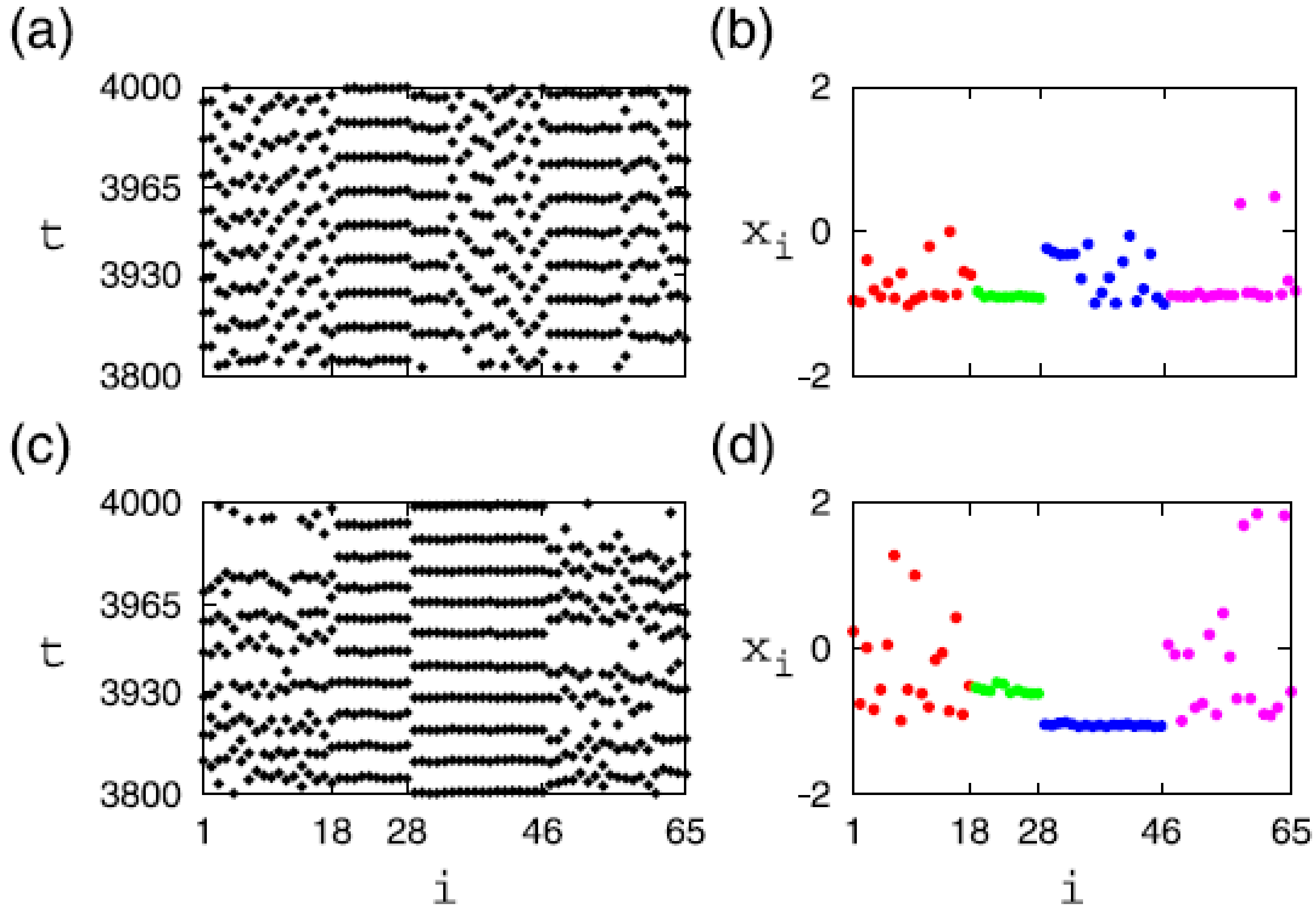


Fig. 3. Space-time plots (left) and snapshot of the variable x (right). (a) and (b) exhibit SC for $\alpha = 0.7$ and $\beta = 0.08$. (c) and (d) show BC for $\alpha = 1.5$ and $\beta = 0.1$.

SC – spiking chimera-like state BC – bursting chimera-like state
Red – visual, magenta – frontolimbic, green – auditory, blue - SM

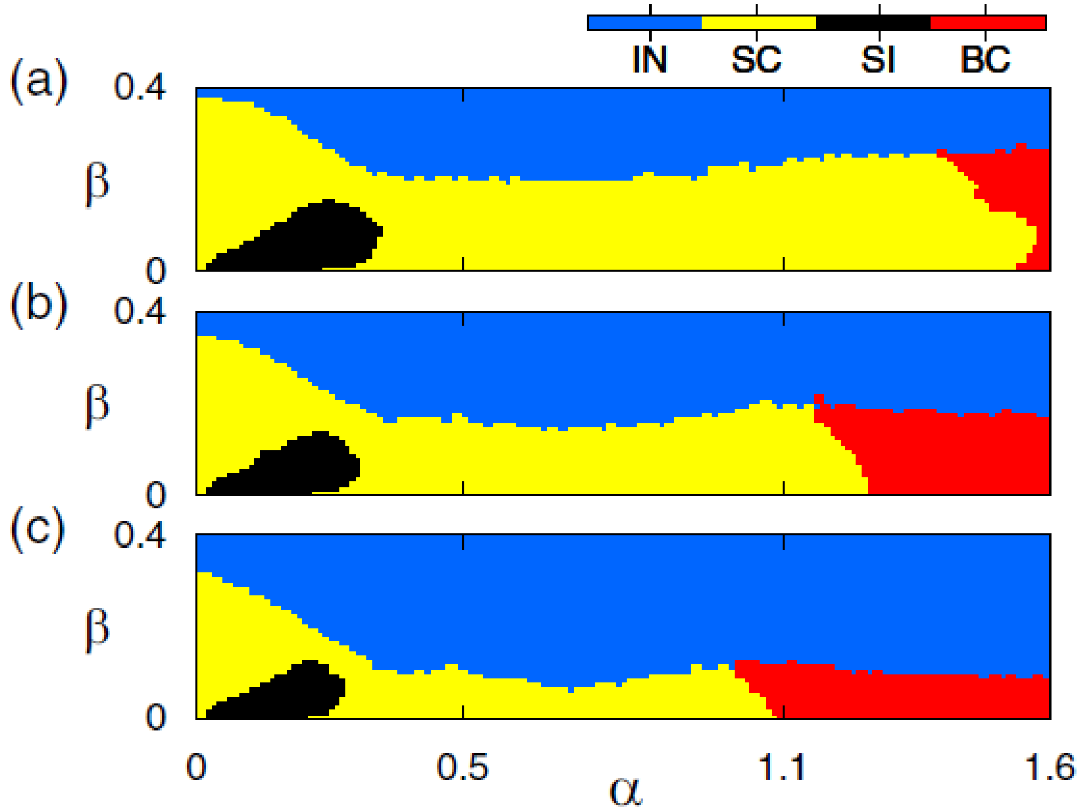


Fig. 4. Parameter spaces $\beta \times \alpha$ for $b = 3.2$, (a) $I_0 = 4.6$, (b) $I_0 = 4.4$, and (c) $I_0 = 4.2$. We see the regions for SC, BC, SI, and IN.

SI – synchronized states, IN – incoherent states (small blocks)

Based on recurrence plots of phase differences