Quantifying Stability of Complex Networks and its Application

Jürgen Kurths

Potsdam Institute for Climate Impact Research
&
Institute of Physics, Humboldt-Universität zu Berlin

POTSDAM INSTITUTE FOR CLIMATE IMPACT RESEARCH
PIK

juergen.kurths@pik-potsdam.de

http://www.pik-potsdam.de/members/kurths/

Main Collaborators:

- **Sabine Auer, Jobst Heitzig, Frank Hellmann, Peter Menck, Paul Schultz** PIK Potsdam & Humboldt Uni Berlin
- **Jinqiao Duan, Meng Zhan, Yayun Zheng** HUST, Wuhan
- **Siyang Leng, Wei Lin, Peng Ji** Fudan Uni
- **Piotr Brzeski, Tomasz Kapitaniak, Przem Perlikowski,** Lodz Univ Technol, Poland
- **Vladimir Klinchuk,** IAP, Nizhny Novgorod

Contents

Infrastructure \ Physiology

Evolving Networks

Network of Networks

Interconnected Networks

Interdependent Networks

Multiplex Networks

Multilayer Networks…

Physiology

network of organs

Challenge:

regarding as complex network – interaction of tipping areas

Cascading (domino-like) or isolated?

Same problem as in physiology

Multistable Dynamics in these networks

Several Regimes possible

What about **Stability** and **Importance** of them?

Power Grids

Intended Solution:

stable synchronized behaviour along the whole network of networks

compared to physiology and climate rather simple problem, **but!**

How to control such networks?

Pinning Control (which nodes?)

Highly Non-trivial Task

Monster blackouts/ dying…

Failing of Control!!!

November 9, 1965

Large Blackout in Northeast of US

> 30 Mio people up to 12 hours without electricity

Speculations about cause

- The Russians are coming (President L. B. Johnson)
- UFOs (observed…)
- …

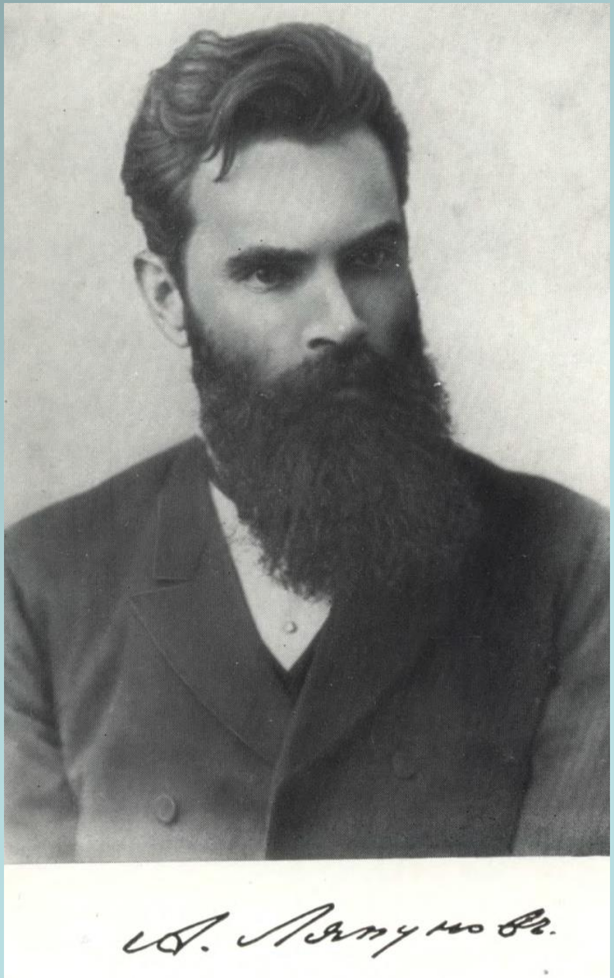
(Highly probable) Cause:

service operators installed a wrong protective relais in a power station near the Niagara Falls…

start 17:16 – cascade effect during 4 minutes monster blackout

Stability

Stability of Dynamical Systems



Alexandr Mikhailovich Lyapunov (1857 – 1918)

- Student of P. L. Chebyshev and friend of A. A. Markov
- Master: On the stability of ellipsoidal forms of equilibrium of rotating fluids (1884) – french translation (1904)
- PhD: The general problem of the stability of motion (1892)
- 1893 – full Prof. Kharkiv Univ
- 1902 – St. Petersburg (followed Chebyshev)

Alexandr Mikhailovich Lyapunov

- Lyapunov was the first to consider modifications necessary in *nonlinear systems* to the linear theory of stability based on linearizing near a point of equilibrium
- The equilibrium x_ε of the system is said to be *Lyapunov stable*, if for every ($\forall\ \varepsilon > 0$) and ($\forall\ t_0$), there exists a $\delta = \delta(t_0, \varepsilon) > 0$ such that,

  if $|x(t_0)-x_\varepsilon| < \delta$, then $|x(t)-x_\varepsilon| < \varepsilon$, for every $t \geq 0$.
- Extension to asymptotical and exponential stability

Stability of Networks

Synchronizability –
Master Stability Formalism

Pecora&Carrol (1998) –

based on **local stability**

Synchronizability – Master Stability Formalism (Pecora&Carrol (1998)

Synchronizability Ratio $R = \lambda_{\mathrm{max}}/\lambda_{\mathrm{min}}$

Stability Interval for coupling strength K

$$K \in I_s = (\alpha_1/\lambda_{\mathrm{min}}, \alpha_2/\lambda_{\mathrm{max}}),$$

Synchronizability condition $R < \alpha_2/\alpha_1$

Stability/synchronizability in small-world (SW) networks

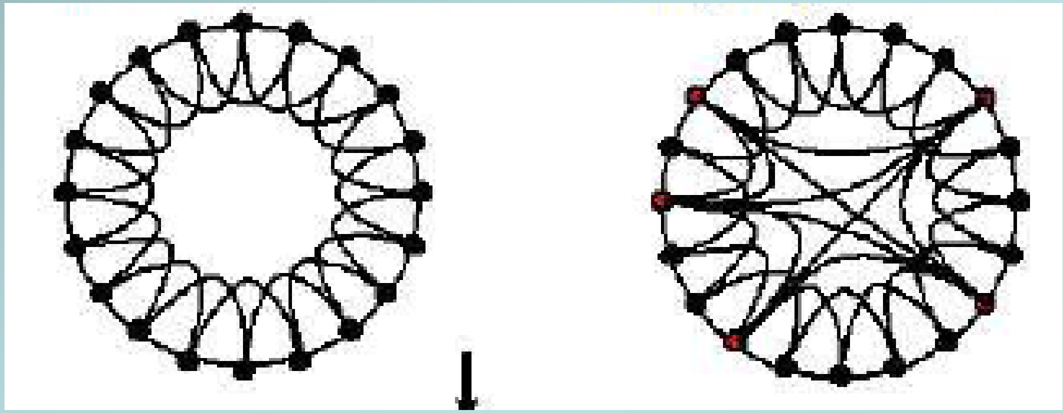
Small-world (SW) networks

(Watts, Strogatz, 1998 – WS-networks)

F. Karinthy hungarian writer –

SW hypothesis (1929)

Small-world Networks



k nearest neighbour connections

Nearest neighbour and a few long-range connections

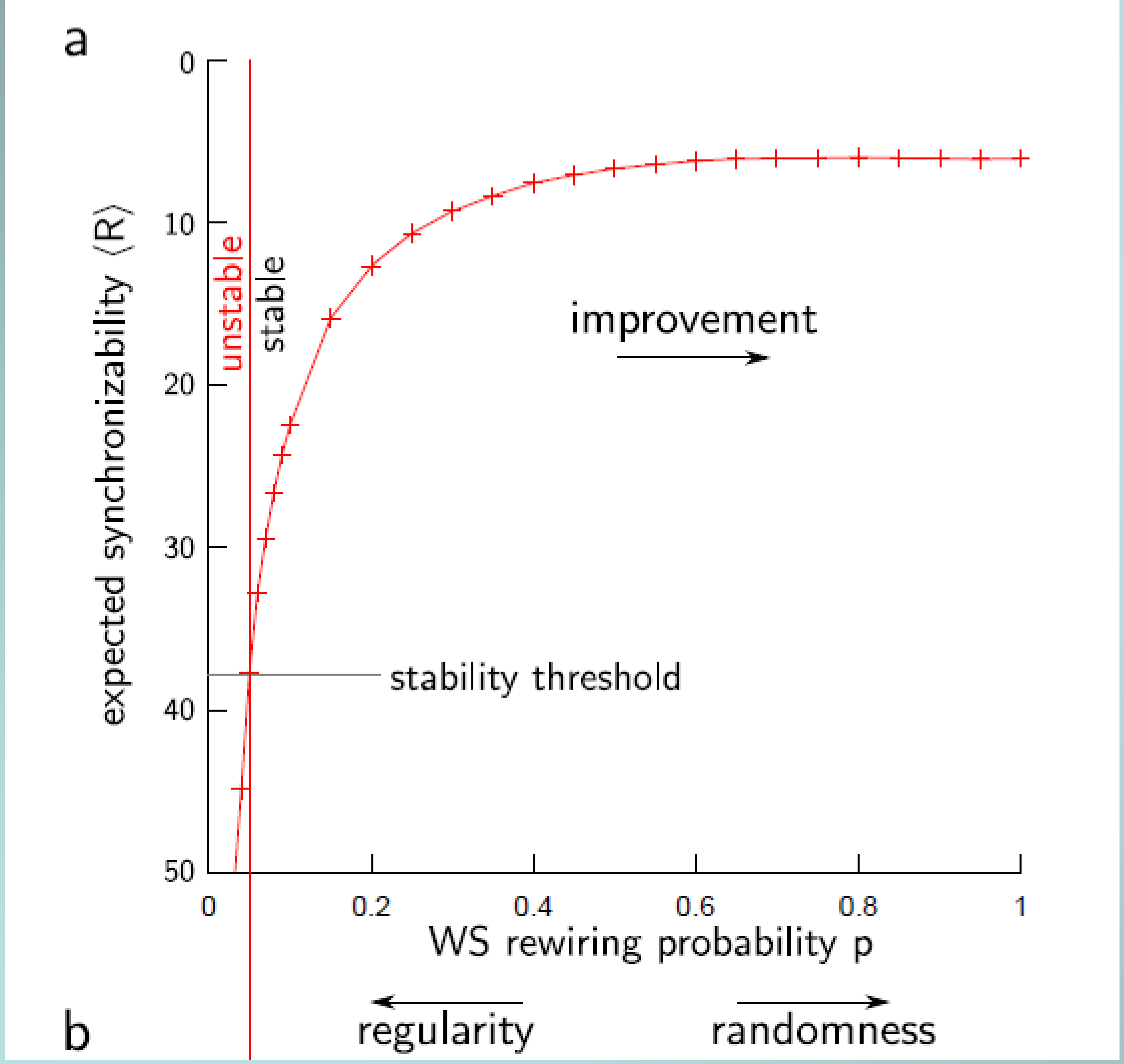
Regular ➔ Complex Topology

$$\dot{\mathbf{r}}_i = \mathbf{F}(\mathbf{r}_i) + K \sum_j A_{ij}[\mathbf{H}(\mathbf{r}_j) - \mathbf{H}(\mathbf{r}_i)] = \mathbf{F}(\mathbf{r}_i) - K \sum_j L_{ij}\mathbf{H}(\mathbf{r}_j),$$

$$\begin{aligned}
\dot{x}_i &= -y_i - z_i - K\sum_{i=1}^{N} L_{ij}x_j \\
\dot{y}_i &= x_i + ay_i \\
\dot{z}_i &= b + z_i(x_i - c)
\end{aligned}$$

Chosen: a = b = 0.2, c = 7.0 ➔ R < 37.88

Chaotic Rössler oscillators, N = 100



Main Result: SW-Network **best synchronizable for most random** SW-networks

Puzzle!

MSF – local stability (Lyapunov stability)

How to go beyond (not only small perturbations)?

Lyapunov Functions?

Network´s Basin Stability

basin volume of a state (regime)

measures likelihood of return to this state (regime)

Nature Physics 9, 89 (2013)

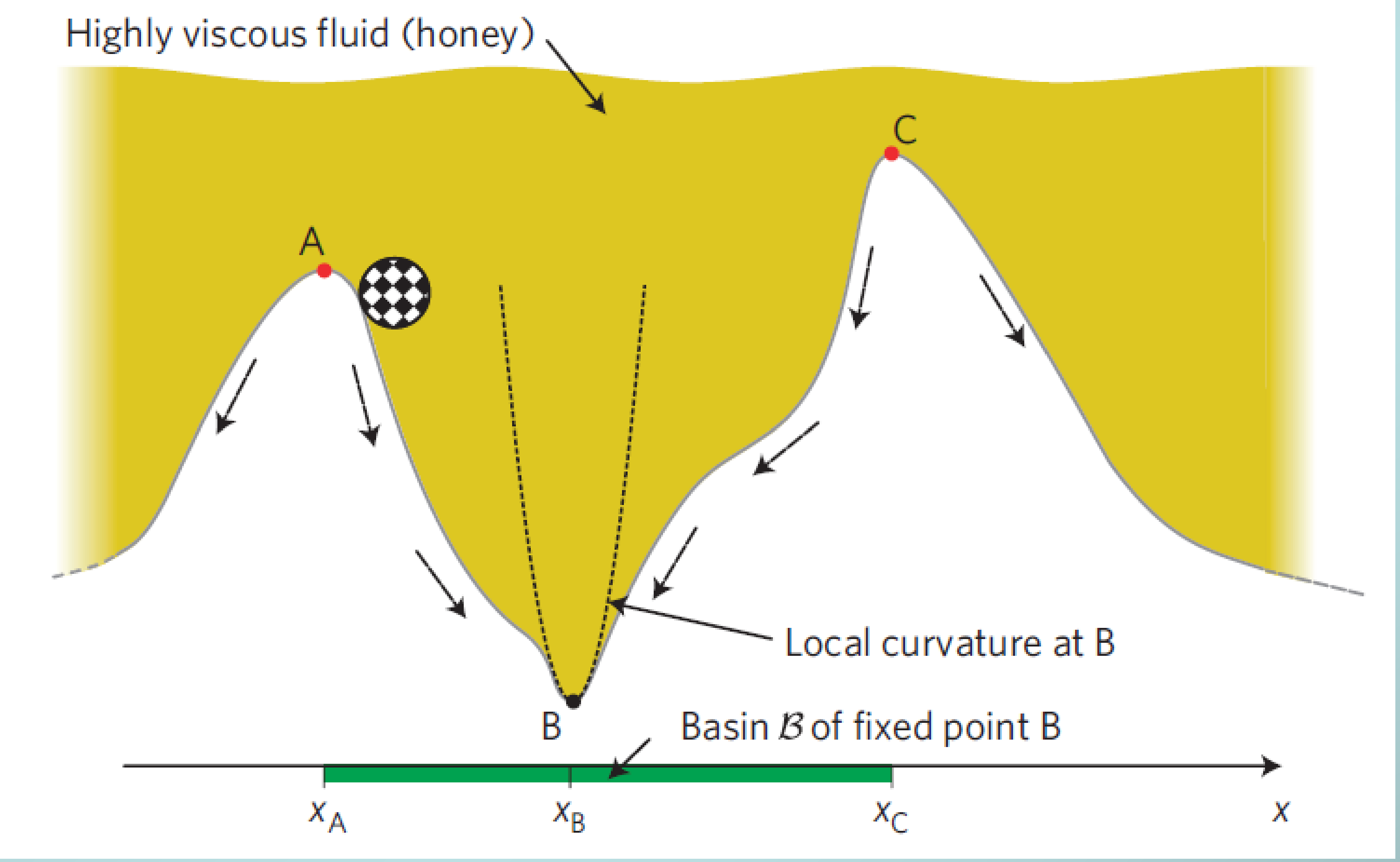


Figure 1 Thought experiment: marble on a marble track. The track is immersed in a highly viscous fluid to make the system's state space one-dimensional. Dashed arrows indicate where the marble would roll from each position. A, B and C label fixed points. Only B is stable. The green bar indicates B's basin of attraction B. If the marble is perturbed from B to a state within the basin, it will return to B. Such perturbations are permissible. Perturbations to states outside the basin are impermissible. The dashed parabola shows the local curvature around B, fitting the true marble track poorly in most of the basin.

Network´s Basin Stability

basin volume of a state (regime) measures the likelihood of

- arrival at this state (regime)
  quantifies its **relevance** (M. Girvan, 2006)
- return to this state after a random perturbation
  quantifies its **stability**
  (Menck, Heitzig, Marwan, Kurths:
  Nat. Phys., 2013)

Normalized Network´s Basin Stability

$\mathcal{B}$ - Synchronous state´s basin of attraction

$$\mathcal{B} = \{\mathrm{x} \in \mathcal{S} \,|\, \Phi_t(\mathrm{x}) \to \mathcal{I}\}$$

$\mathcal{Q}$ - Subset of state space S covering the system´s (weak) attractor

$$S_{\mathcal{B}\cap\mathcal{Q}} = \mathrm{Vol}(\mathcal{B} \cap \mathcal{Q})/\mathrm{Vol}(\mathcal{Q}) \in [0, 1]$$

Normalized Basin Stability

Bernoulli-like experiment

- T experiments (different initial conditions – randomly distributed)
- M states converge to $\mathcal{I}$
- Estimate M / T

  ➔ standard error $e := \frac{\sqrt{S_{\mathcal{B}}(1 - S_{\mathcal{B}})}}{\sqrt{T}}$

- T=500 ➔ error < 0.023

  No dependence on dimension!!!

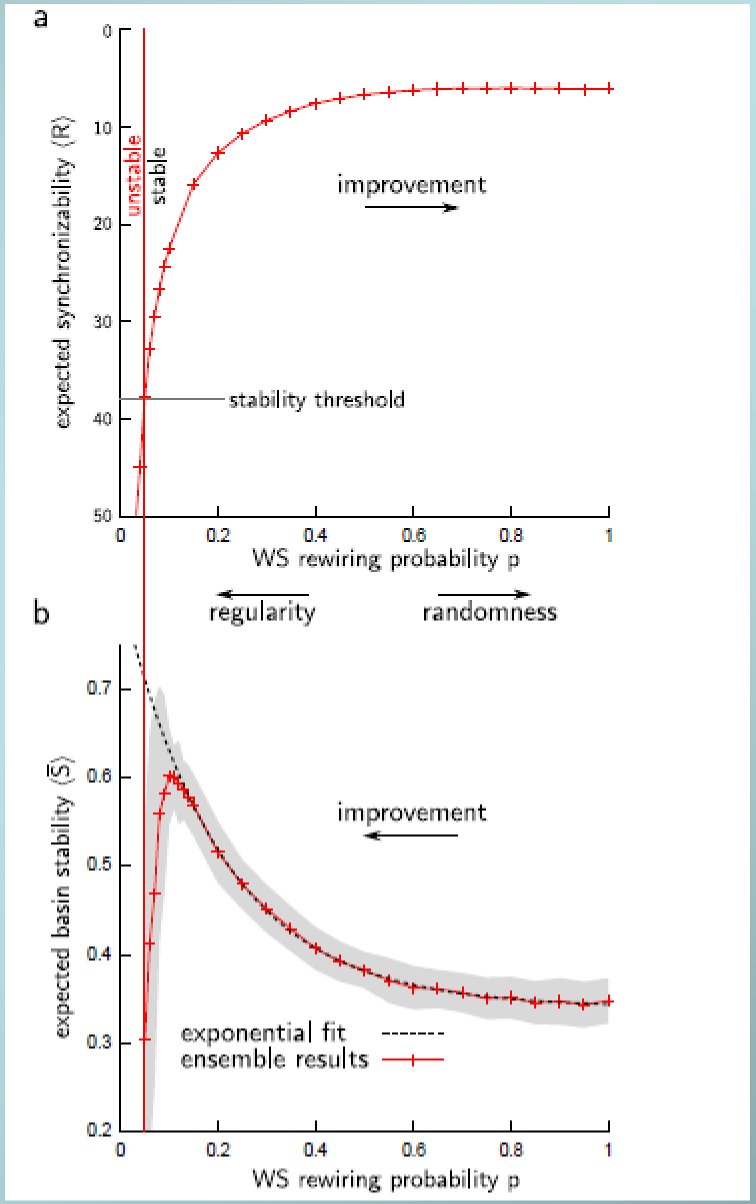
Synchronizability and basin stability inWatts-Strogatz (WS) networks of chaotic oscillators.

a: Expected synchronizability R versus the WS model's parameter p.

The scale of the y-axis was reversed to indicate improvement upon increase in p.

b: Expected basin stability S versus p. The grey shade indicates one standard deviation.

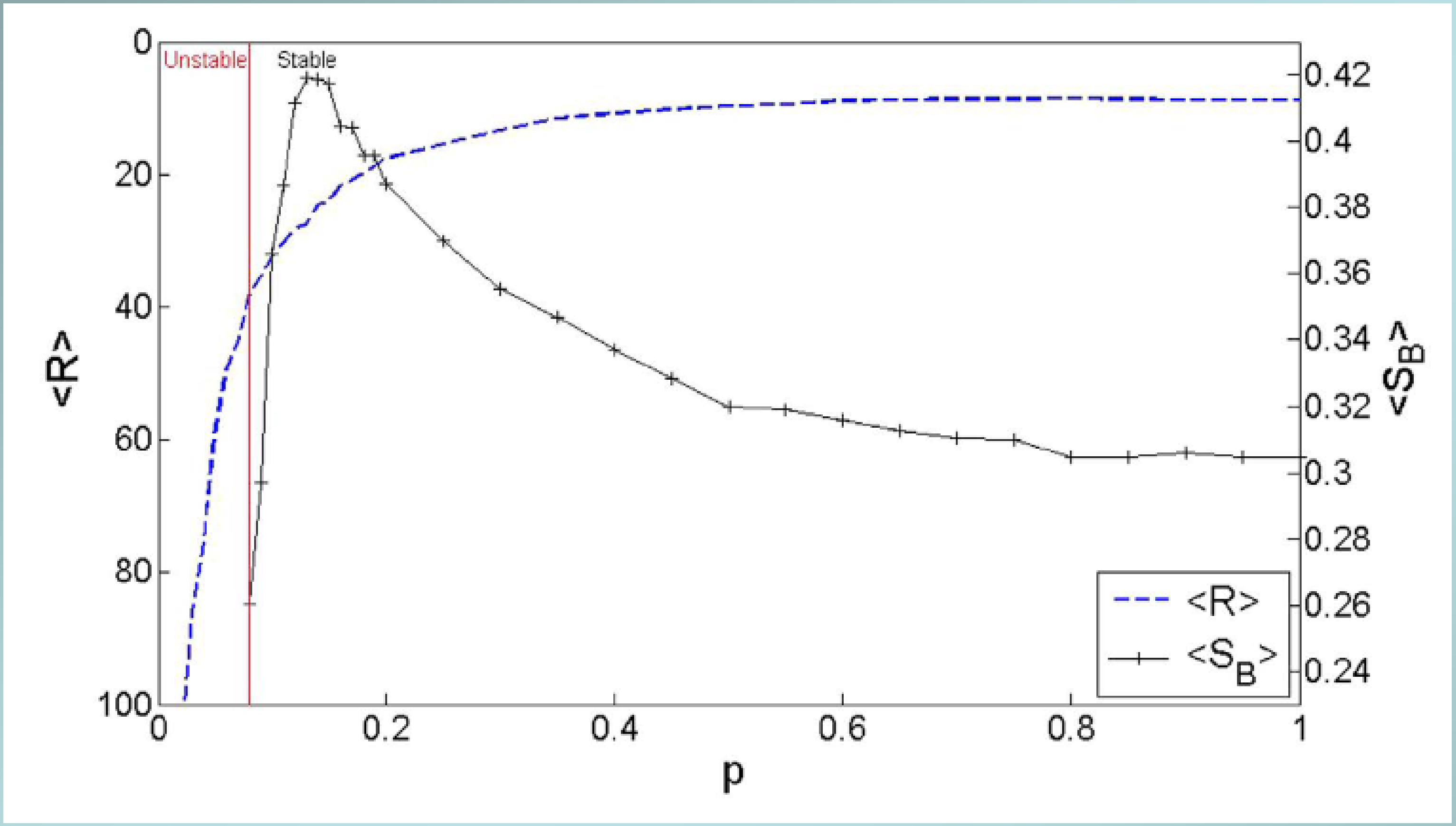
The dashed line shows an exponential fitted to the ensemble results for p > 0.15. Solid lines are guides to the eye. The plots shown were obtained for N = 100 oscillators of Roessler type, each having on average k = 8 neighbours. Choices of larger N and different k produce results that are qualitatively the same.



Extension to delay-coupled systems

$$\dot{X}_i(t) = F[X_i(t)] - \sigma \sum_{j=1}^{N} g_{ij} h[X_j(t-\tau)],$$

Scient. Rep., 2016



SW network, N = 100, chaotic Roessler oscillators, 6 neighbours each (in average) $\tau = 0.4.$

Other Approaches

- Basin stability refers to **asymptotic** behaviour and requires **multistability**
- In many applications (cybersystems, power grids, brain, climate…) **transient** behaviour more important
- Apply concept of survivability

➔**Basin of Survival**

Scient. Rep. 2016

Desirable region $X^+ \subset X$

Survivability S(t):

Fraction of trajectories starting at X^+ and staying within X^+ the whole time [0 , t]

t-time basin of survival X_t^S

$$S(t) = \frac{\mathrm{Vol}(X_t^S)}{\mathrm{Vol}(X^+)}$$

Application: Power Grids

Power Grid Model

$$\dot{\theta}_i = \omega_i$$

$$\dot{\omega}_i = -\alpha_i \omega_i + P_i - \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j)$$

θ_i and ω_i denote phase and frequency of the generator at node i

Node i net generator if $P_i > 0$

Node i net consumer if $P_i < 0$

α_i - damping constant

P_i - net power input

Only two solutions (regimes):

1) synchronized (wanted)

2) periodic (unwanted)

Main Question:
How stable is the synchronized regime?

$$\omega_i = 0, \; \dot{\omega}_i = 0$$

Stability even in case of large perturbations at one node

➔Concept of basin stability

Nature Commun. 5, 3969 (2014)

Single Node Basin Stability:

Perturb Initial Conditions **only** at Node i

$$\begin{pmatrix} \theta_1(0) \\ \omega_1(0) \\ \vdots \\ \theta_i(0) \\ \omega_i(0) \\ \vdots \\ \theta_N(0) \\ \omega_N(0) \end{pmatrix} = \begin{pmatrix} \theta_1^s \\ 0 \\ \vdots \\ \theta_i^s \\ 0 \\ \vdots \\ \theta_N^s \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \Delta\theta_i \\ \Delta\omega_i \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Application to the Scandinavian Power Grid

Figure 4: **Northern European Power Grid.** The grid has $N = 236$ nodes and $E = 320$ transmission lines. The load scenario was chosen randomly, with squares (circles) depicting $N/2$ net consumers with $P_i = -P$ (net generators with $P_i = +P$). The colour scale indicates how large a node's basin stability S_i is. Insets I-III show re-computed basin stability values after 27 lines have been added in order to 'heal' dead trees (see Methods). New lines are coloured red. Our simulation parameters, $\alpha = 0.1$, $P = 1$, and $K = 8$, imply the simplifying assumptions that all generators in the grid are of the same making and that all transmission lines are of the same voltage and impedance. These assumptions enable us to focus on the effects of the (unweighted) topology. For details, see Methods.

First Conclusions

- Concept of basin stability enables important new insights and principles for the design of (Smart) Power Grids
- Dead ends and dead trees strongly diminish stability (**trouble makers**) ➔ to be avoided
- „Healing“ dead ends by addition of a few transmission lines enhances substantially stability
- For the Scandinavian power grid: addition of 27 lines (8 % of the total) suffice to substantially improve stability – rather low-cost solution)

Power grids with losses

$$H_i\ddot{\phi}_i = P_i - D_i\dot{\phi}_i - \sum_{j=1}^{n} P_{ij} \ ,$$

$$P_{ij} = K_{ij}\Big(\sin(\alpha_{ij}) + \sin(\phi_i - \phi_j - \alpha_{ij})\Big)$$

Complex Admittance $Y_{ij} = -iK_{ij}\exp(i\alpha_{ij})$

X reactance
R resistance

$$Y_{ij} = \frac{1}{R_{ij} + iX_{ij}}$$

Loss-free correct?

- In most power grid studies considered (as above):

  $\alpha = 0$ (because $R = 0$)

- But in reality:

  $\alpha = 0.24$ – high-voltage power grids

  $\alpha = 1.4$ – medium voltage…

Crucial question:

Which consequences have losses for solutions and stability?

New multistability occurs

- Shift of limit cycle:

$$\omega'_{\mathrm{lc}} = \omega_{lc} - \frac{K}{D} \sin(\alpha)$$

- Strongly change the basin structure of the solitory (periodic) solution
- Even flips signs of rotation ➔ **exotic** oscillates in opposite dir. **solitary state**
- Further solutions appear

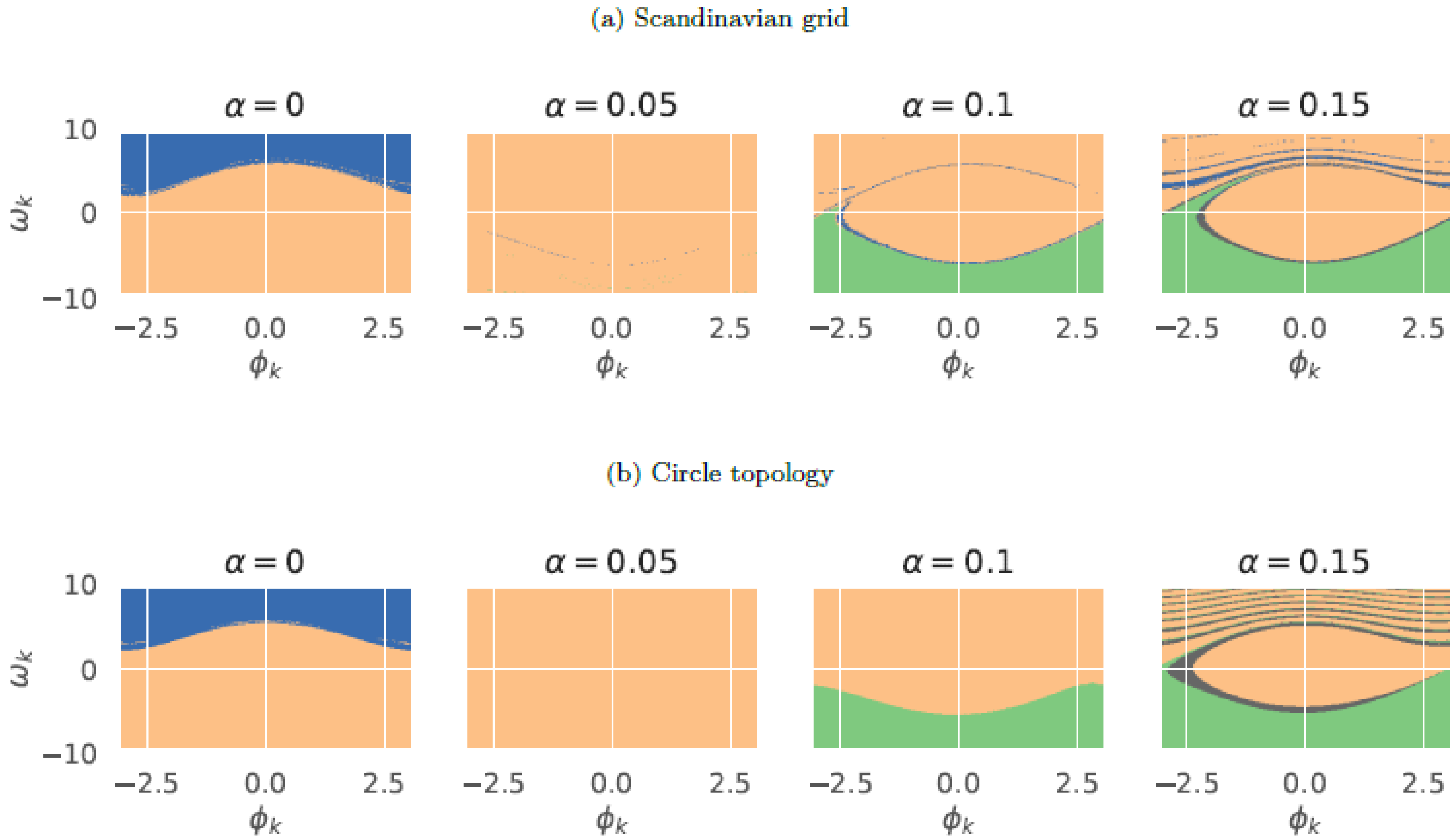


FIG. 2: **Phase space cross sections.** Cross section of the phase space corresponding to phase ϕ_k and phase velocity ω_k of a randomly chosen node of **a** the Scandinavian power grid and **b** the circle topology (both with standard parametrisation and control, see Methods). Each point belongs to the sync basin (■), the basin of a solitary state rotating naturally (■) or in the basin of an exotic solitary state (■). Other asymptotic states are marked in grey (■). Further parametrisations are given in SI.

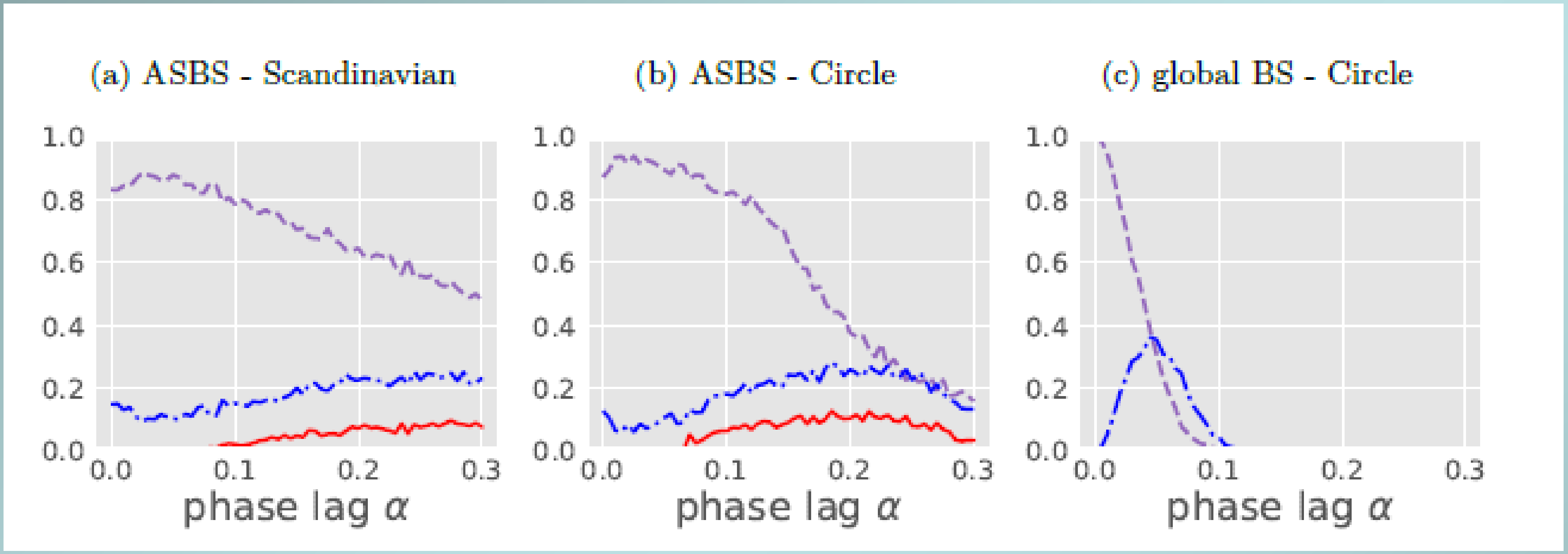


FIG. 3: **Basin stability.** The top row shows the average single node basin stability ASBS (**a, b**) and the global basin stability BS (**c**) of three types of asymptotic regimes: synchronisation (- - -), exotic solitaries (——) and the union of normal and exotic solitaries (-·-·-). Simulations were performed with standard parametrisation and control

Challenges for power grids

- Early forecasting of **extreme (climate) events** and track (from others)
- Estimating **risk** for impacts – failure of synchronization, unknown dynamics may occur
- **Mitigation** procedures – problems of design and control – new approaches necessary
- Illustration for one example

Problems

Strongly (extreme) multistable systems

Identification of all IMPORTANT regimes and then determining their basins

How to (roughly) estimate BS from experimental data?

Active vs. Passive experiments

- **Active**: initial conditions and parameters can be (easily) modified, e.g. electr. circuits, lasers, mechanical systems

- **Passive**: not much changes of the experiments possible/ allowed (physiology, climatology)
  BUT
  climate: looking into the past (palaeo)
  physiology: different treatment/ environment of patients

Our Papers

- Nature Physics 9, 89 (2013)
- Nature Communication 5, 3969 (2014)
- Comm. Comp. Inform. Sc. 438, 211 (2014) . EPJ ST 223, 2593 (2014)
- New J. Phys. 16, 125001 (2014) Phys. Rev. E 90, 022812 (2014)
- Phys. Rev. Lett. 112, 114102 (2014) Phys. Rev. E 92, 042803 (2015)
- Scient Rep. 5, 16196 (2015) Meccanica (2016)
- New J. Phys. 18, 013004 (2016) CHAOS 27, 127003 (2017)
- Scient. Rep. 6, 21449 (2016) New J. Phys. 19, 023005 (2017)
- Scient. Rep. 6, 29654 (2016) New J. Phys. 19, 033029 (2017)
- Physics Rep. 610, 1 (2016) Scient. Rep. 7, 9336 (2017)
- CHAOS 26, 073117 (2016) CHAOS 28, 043102 (2018)
- CHAOS 30, 013110 (2020) Phys Rev Res 2, 023409 (2020)
- Nature Communications 11, 592 (2020)
- New J. Phys 24, 043041 (2022) New J. Phys. 24, 053019 (2022)
- IEEE Trans Energiy Conversion 37, 1428 (2022)
- IEEE ACCESS 10, 1289 (2022)