Reconstruction of Network Connectivity from Observations

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Part I: **continuous time signals, appropriate for data estimation**

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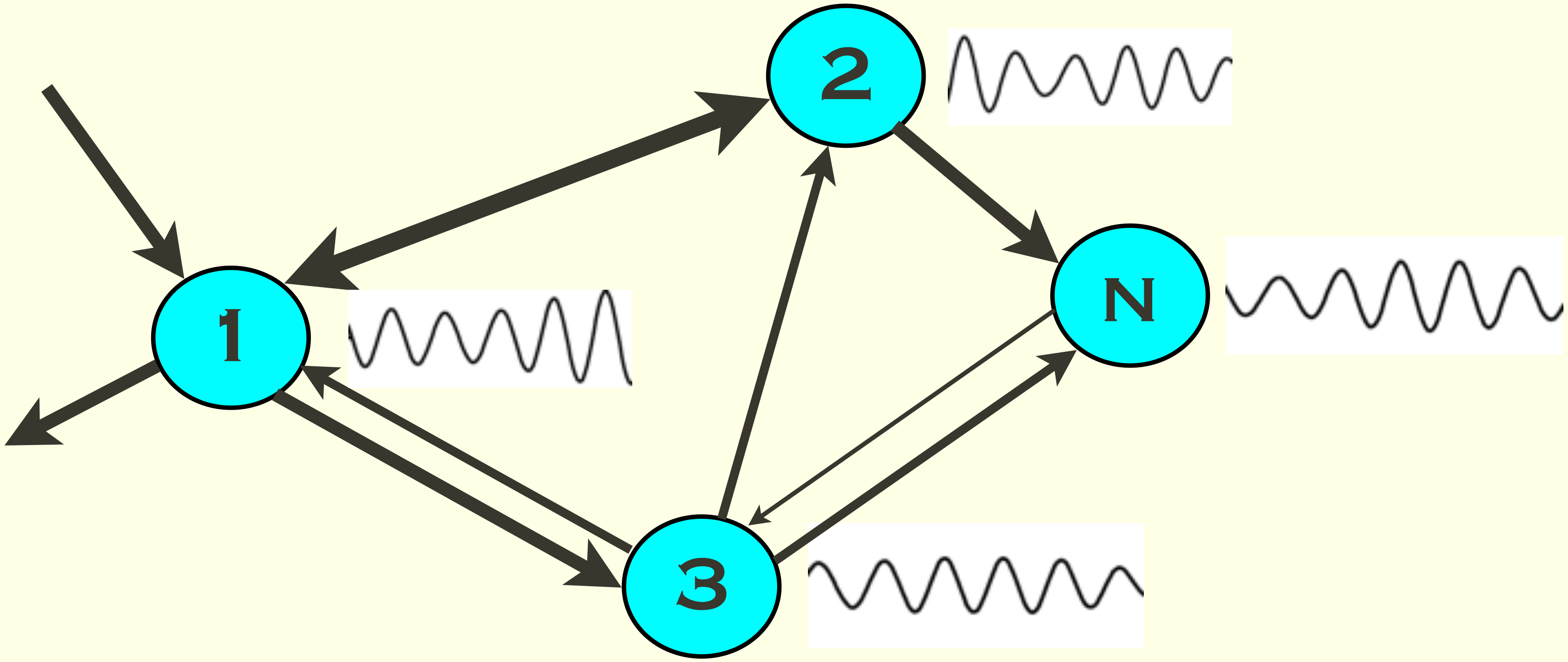
Part II: **spike data (point processes)**

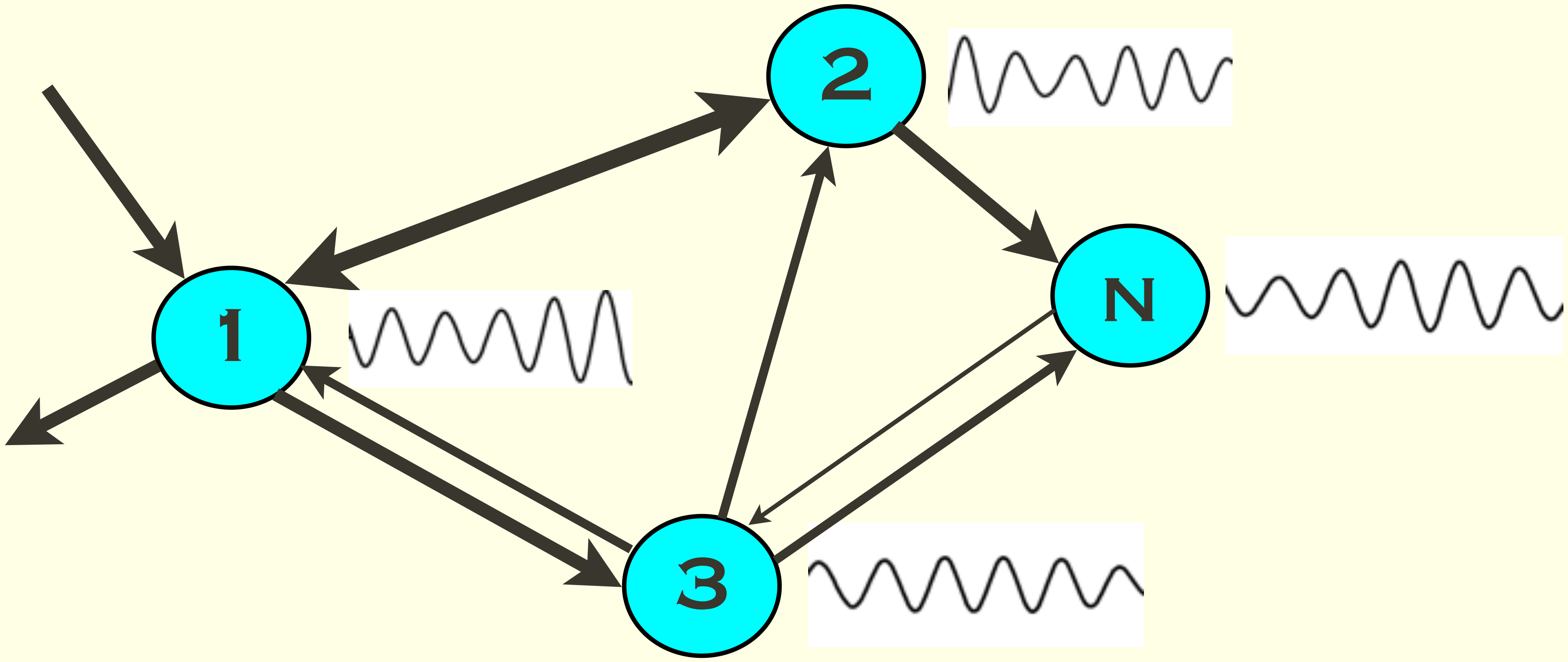
Rok Cestnik

Formulation of the problem



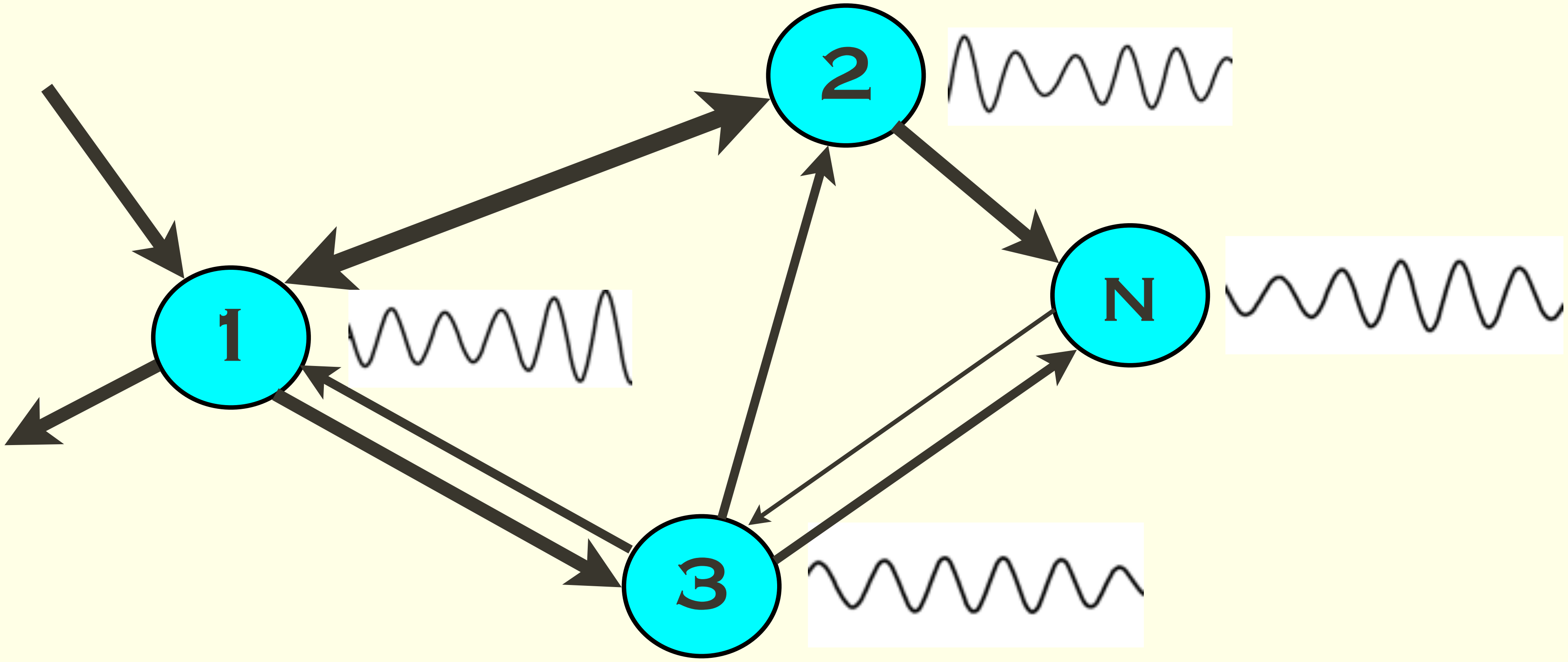
- Data: we have oscillatory signals coming from several ***weakly coupled self-sustained oscillators***
- Our goal: to say as much as possible about the systems and their interaction
- Particular problem: to reconstruct ***directional connectivity***
- What kind of connectivity do we detect?
  - this will be discussed in detail later

Formulation of the problem: assumptions



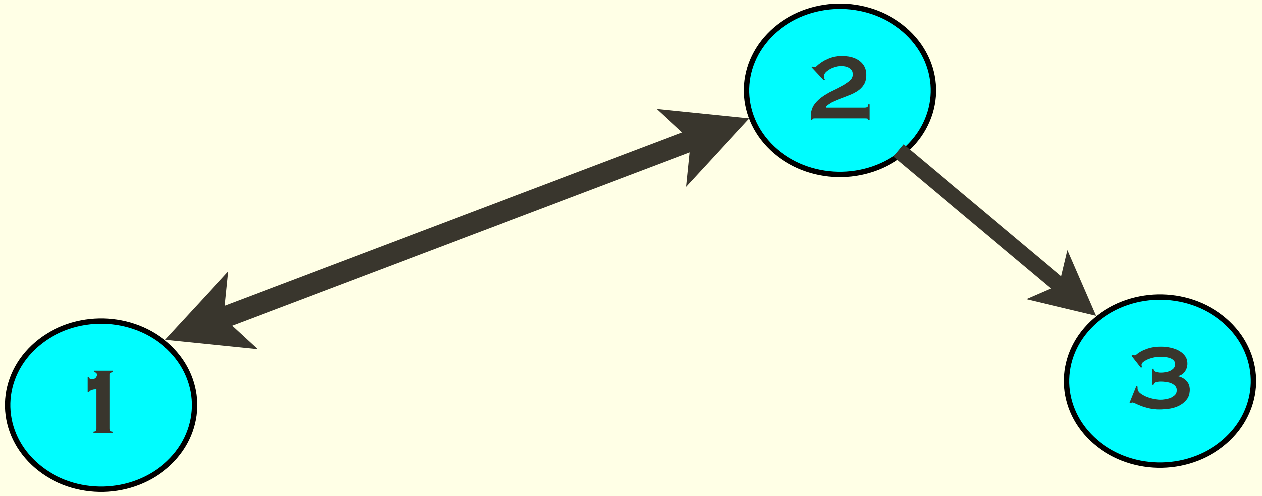
- Assumption 1: all units are observed
- Assumption 2: the units are **self-sustained** oscillators
- Assumption 3: the interaction between the units is not too strong
- Assumption 4: signals are good for **estimation of phases**

Connectivity of an Oscillator Network



- Data: we have oscillatory signals coming from several ***weakly coupled self-sustained oscillators***
- Problem: to reconstruct ***directional connectivity***
- What kind of connectivity do we detect?
  Structural vs effective vs functional connectivity

Structural connectivity

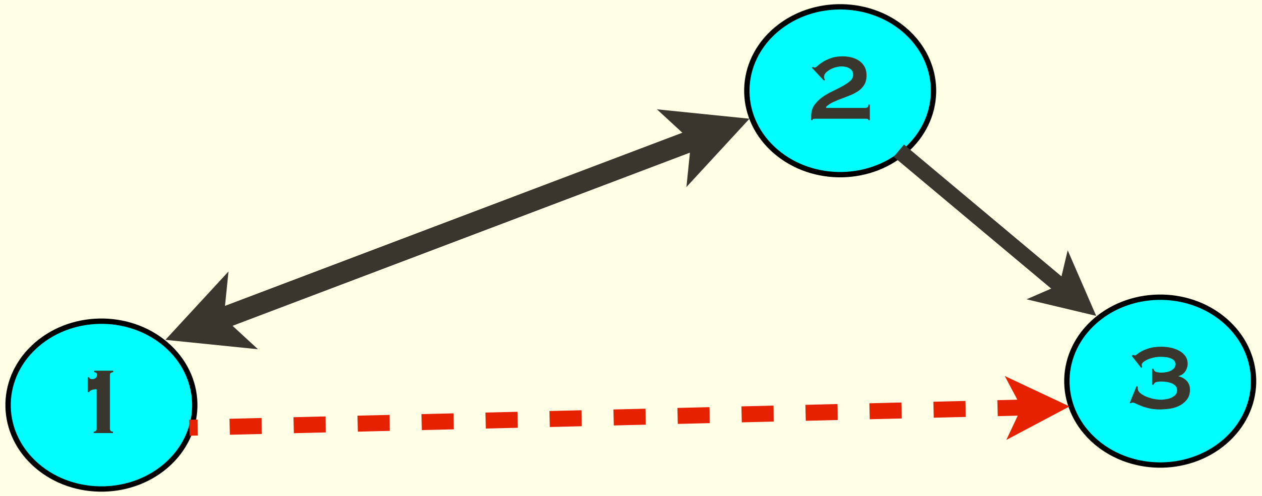


- Real physical connection: resistor, optical fiber...
  Biological system: anatomical connection, e.g., via synapses
- Mathematically, e.g., for the 2nd node:

$$\dot{\mathbf{x}}_2 = \underbrace{\mathbf{G}_2(\mathbf{x}_2)}_{\text{autonomous dynamics}} + \varepsilon \underbrace{\mathbf{H}_2(\mathbf{x}_2, \mathbf{x}_1)}_{\text{coupling function}}$$

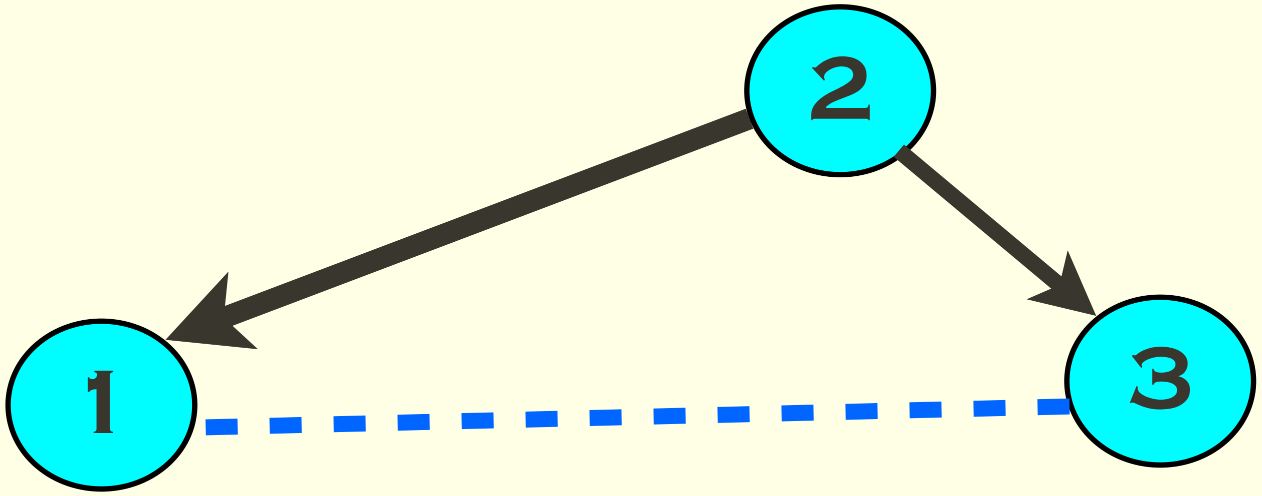
Remark: “coupling” = “physical connection”

Effective phase connectivity



- Nodes 1 and 3 are not physically connected, but phase dynamics of node 3 may depend on the state of node 1. Then, nodes 1, 3 are ***effectively connected*** (unidirectionally)
- Structural connectivity $\neq$ effective phase connectivity

Functional connectivity



- Nodes 1 and 3 are not physically connected, but they may be correlated or synchronized due to the common drive 2

  ⟹ Nodes 1, 3 are functionally connected

- Notice: (1) functional connectivity is not directed

  (2) functional connectivity is only loosely related to the structural and effective ones

We quantify the

effective phase connectivity

by reconstructing the model of phase

dynamics from data

Namely, we perform:

- Protophase estimation
- Protophase-to-phase transformation
- Reconstruction of coupling functions
- Analysis of coupling functions

Network of coupled oscillators

- Individual oscillator: $\dot{\mathbf{x}}_k = \mathbf{G}_\mathrm{k}(\mathbf{x}_\mathrm{k})$
  - limit cycle, parameterized by phase φ_k
  - phase grows linearly with time: $\dot{\varphi}_k = \omega_k = \mathrm{const}$
- A network of N coupled oscillators

$$\dot{\mathbf{x}}_k = \mathbf{G}_\mathrm{k}(\mathbf{x}_\mathrm{k}) + \varepsilon \mathbf{H}_k(\mathbf{x}_1, \mathbf{x}_2, \ldots)$$

- If $\mathbf{x}_l$ enters the equation for $\mathbf{x}_k$ then there is a direct structural connection $l \to k$
- If $\mathbf{H}_k = \sum_{j \neq k} \mathbf{H}_{kj}(\mathbf{x}_\mathrm{k}, \mathbf{x}_\mathrm{j})$ then coupling is pairwise

(We consider only this case)

- If there are terms $\mathbf{H}_{kjl}(\mathbf{x}_\mathrm{k}, \mathbf{x}_\mathrm{j}, \mathbf{x}_\mathrm{l})$: cross-coupling

Weak coupling: Phase description

- Weak coupling, no synchrony: motion on the ***N-torus*** in the phase space of the full system

- This motion can be parameterized by *N* phases:

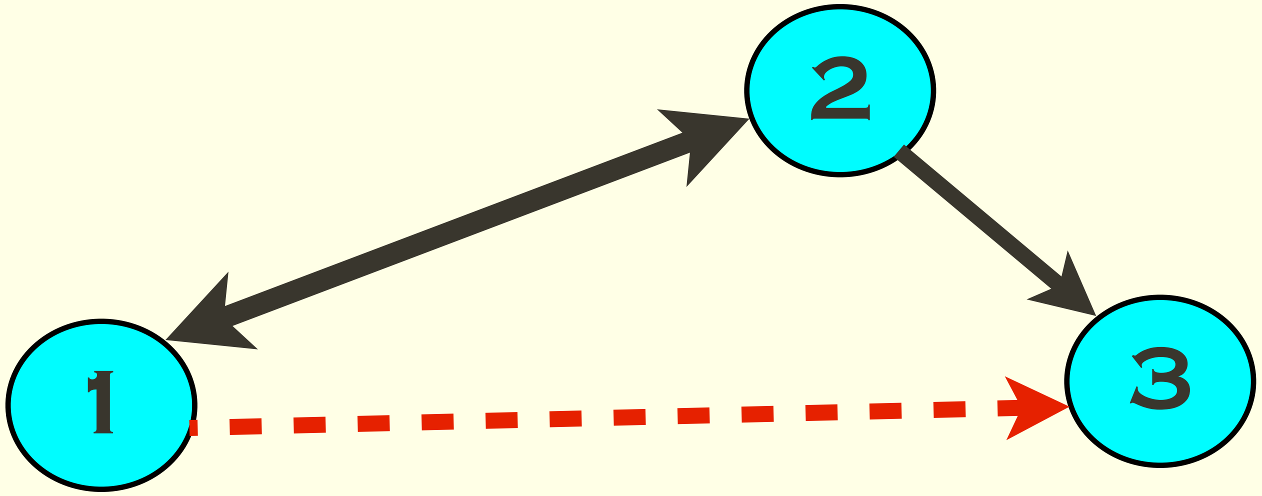
$$\dot{\varphi}_k = \omega_k + q_k(\varphi_1, \varphi_2, \ldots), \quad k = 1, \ldots, N$$

- New coupling functions q_k can be obtained by a perturbative reduction (Kuramoto 84):

$$q_k(\varphi_1, \varphi_2, \ldots) = \varepsilon q_k^{(1)}(\varphi_1, \varphi_2, \ldots) + \varepsilon^2 q_k^{(2)}(\varphi_1, \varphi_2, \ldots) + \ldots$$

- Pairwise coupling in the full system:
  - first-order approximation: pairwise terms like $\varepsilon q_{kl}^{(1)}(\varphi_k, \varphi_l)$
  - high-order approximation: ***terms, depending on many phases***, not only on the phases of directly coupled nodes

Effective phase connectivity



- Nodes 1 and 3 are not physically connected, but phase dynamics of node 3 may depend on the state of node 1. Then, nodes 1, 3 are ***effectively connected*** (unidirectionally)

$$\dot{\varphi}_3 = \omega_3 + \varepsilon q_3^{(1)}(\varphi_2, \varphi_3) + \varepsilon^2 q_3^{(2)}(\varphi_1, \varphi_2, \varphi_3)$$

- Structural connectivity $\neq$ effective phase connectivity

There is no effective phase connection 3⟶1 !

Coupling functions and quantification of interaction

We reconstruct the coupling functions in terms of Fourier coefficients, using LMS fit:

$$\frac{d\varphi_k}{dt} = \omega_k + q_k(\varphi_1, \varphi_2, \ldots, \varphi_N)$$

$$= \sum_{l_1,\ldots,l_N} \mathcal{F}^{(k)}_{l_1,\ldots,l_N} \exp\left(il_1\varphi_1 + il_2\varphi_2 + \ldots + l_N\varphi_N\right)$$

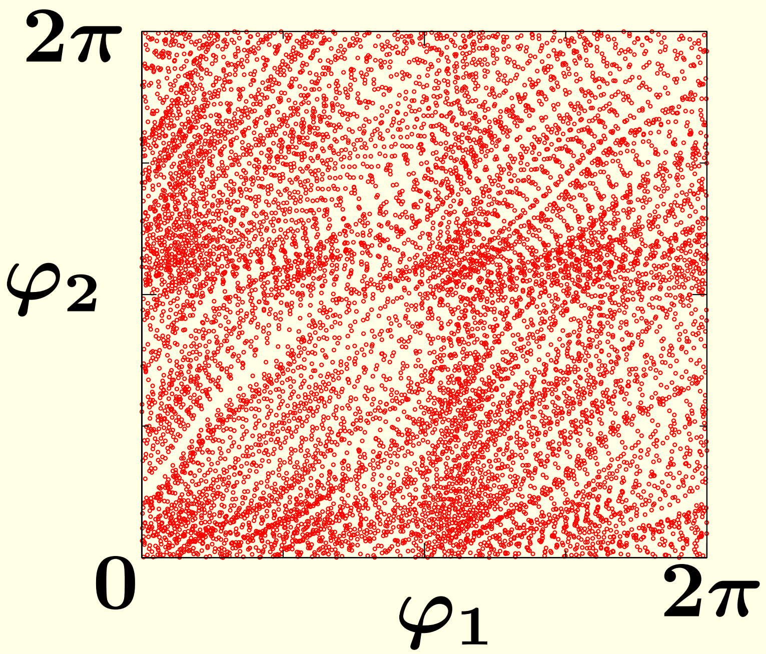
Norm of the coupling function q_k quantifies effect of the rest of the network on oscillator k

Action of particular oscillator $j \to k$

Partial norm $\mathcal{N}^2_{k\leftarrow j} = \sum_{l_k,l_j\neq 0} \left|\mathcal{F}^{(k)}_{0,\ldots,l_k,0,\ldots,l_j,0,\ldots}\right|^2$

Numerical problem

- Two coupled oscillators: to reconstruct the coupling function we need enough data points to cover the square $0 < \varphi_{1,2} \le 2\pi$



- Three coupled oscillators: we need enough data points to cover the cube $0 < \varphi_{1,2,3} \le 2\pi$
- *N* coupled oscillators: we need enough data points to cover the hypercube.... It is not feasible!

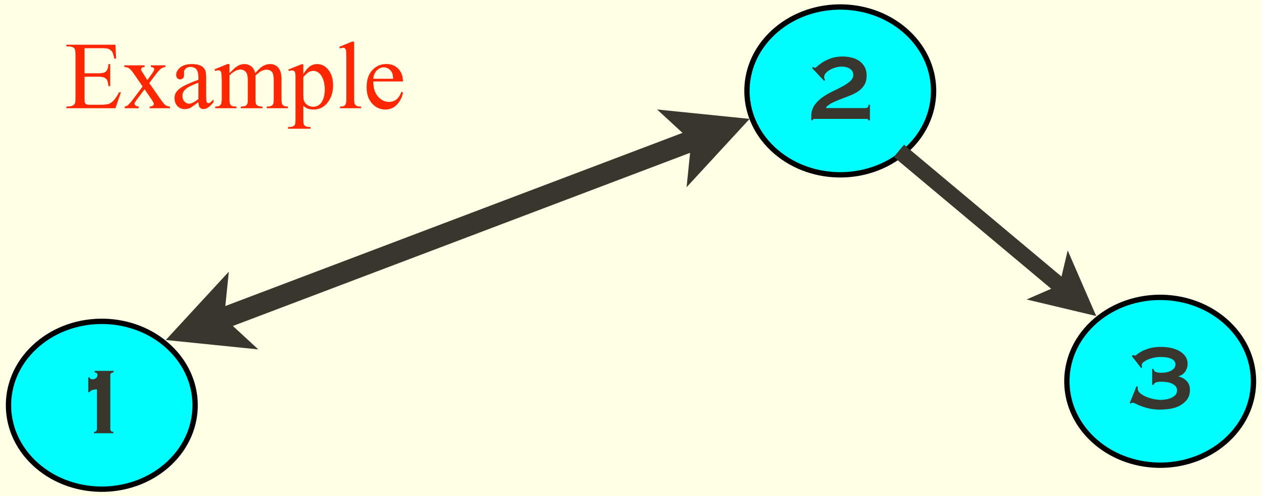
Typically: pairwise analysis. We suggest an analysis by triplets.

Partial phase dynamics

Pairwise analysis: we fit the function of two phases, ignoring all others: $\dot{\varphi}_k = \omega_k + q_{kj}(\varphi_j, \varphi_k)$

Norm $\mathcal{P}_{k\leftarrow j} = ||q_{kj}||$ quantifies link $k \leftarrow j$

Example



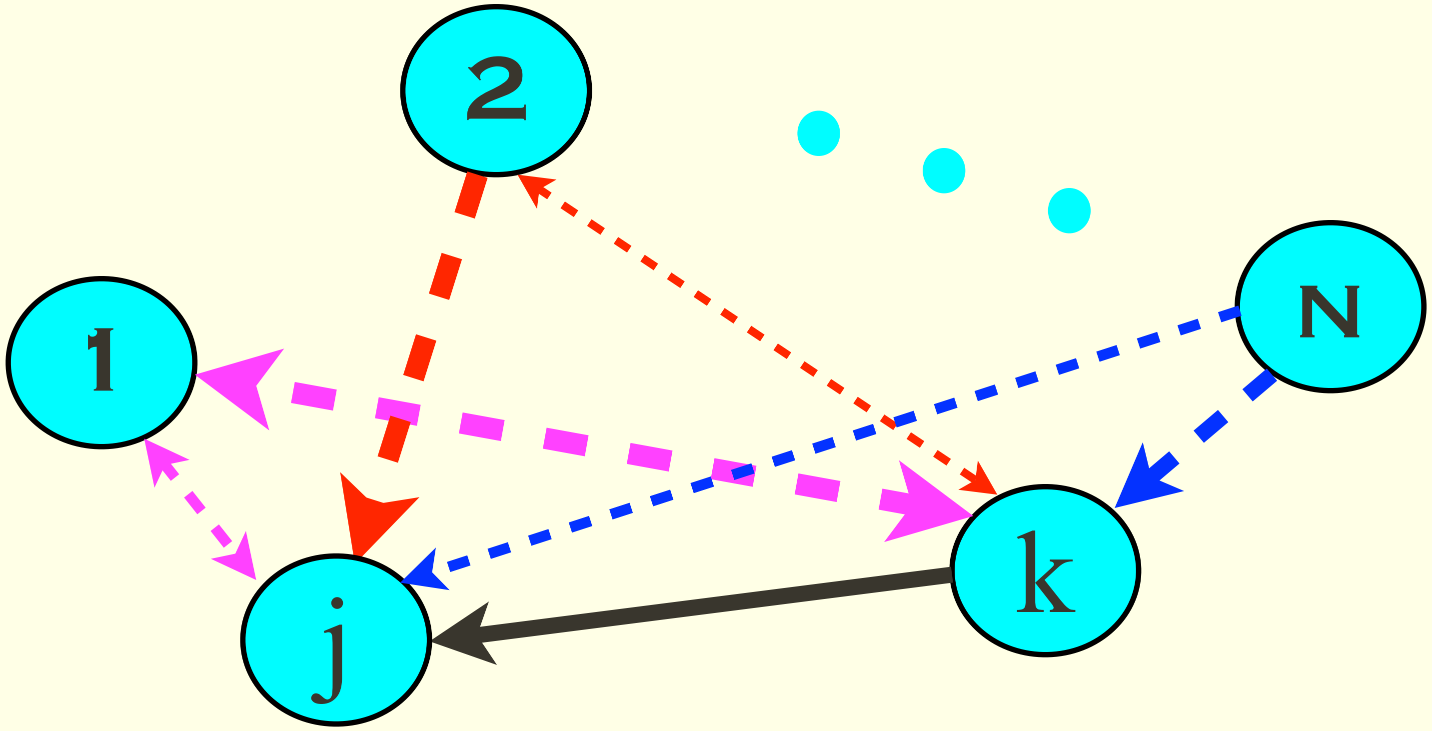
Pairwise analysis yields spurious connection $1 \to 3$

Triplet analysis yields correct connectivity (Kralemann et al 2011)

What to do for networks with *N*>3?

Triplet analysis of networks with *N*>3

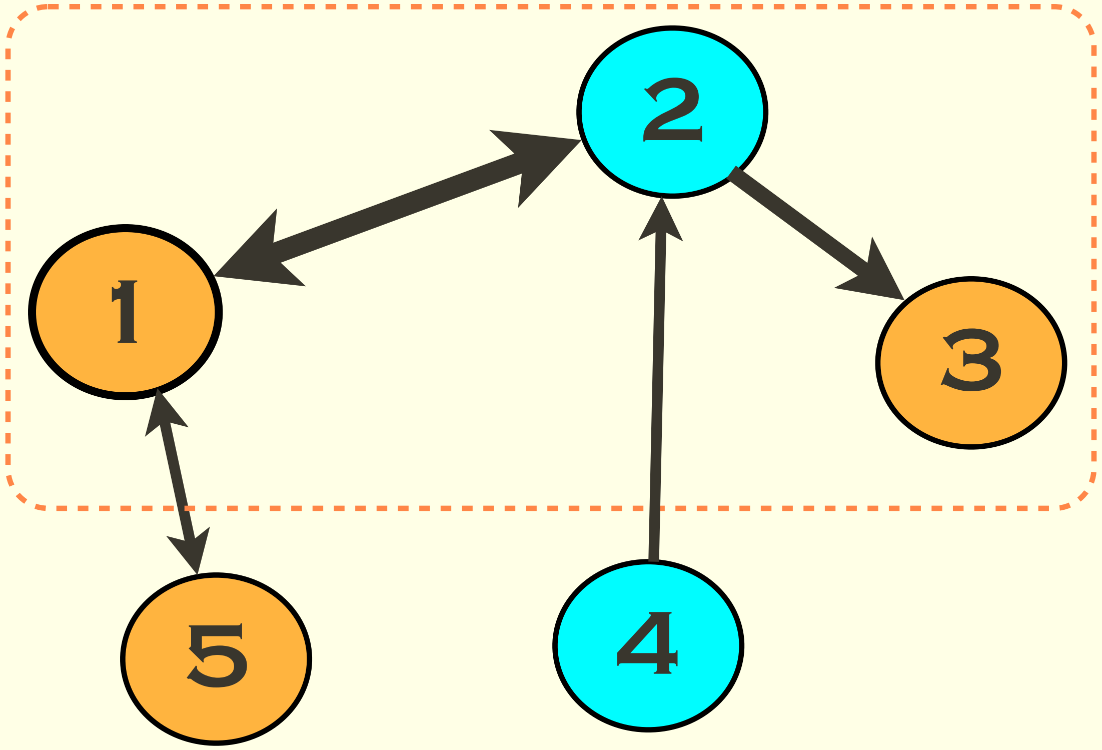
We reconstruct $\dot{\varphi}_j = \omega_j + q_{jkm}(\varphi_j, \varphi_k, \varphi_m)$ for all m



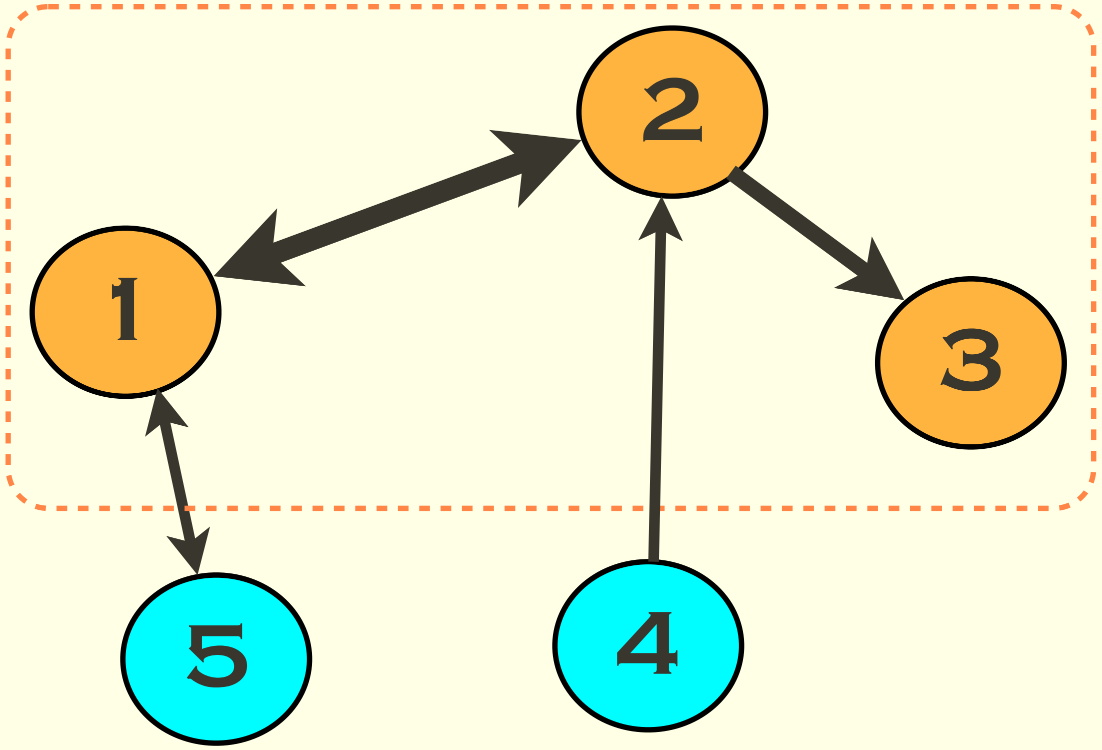
From each triplet we obtain partial norm: $\tilde{\mathcal{T}}^2_{j\leftarrow k}(m) = \sum_{l_j, l_k \neq 0} \left| \mathcal{F}^{(j)}_{l_j, l_k, 0} \right|^2$

We suggest to take $\mathcal{T}_{j\leftarrow k} = \min_m \tilde{\mathcal{T}}_{j\leftarrow k}(m)$ as the final triplet-based measure of the binary effective connectivity

Triplet analysis of networks with *N*>3



Triplet {1,3,5} yields spuriously large term $\mathbf{1} \to \mathbf{3}$, because φ_1, φ_3 are correlated due to node 2



Triplet {1,2,3} correctly explains correlation of φ_1, φ_3 and yields a small value for the link $\mathbf{1} \to \mathbf{3}$

Example: three van der Pol oscillators

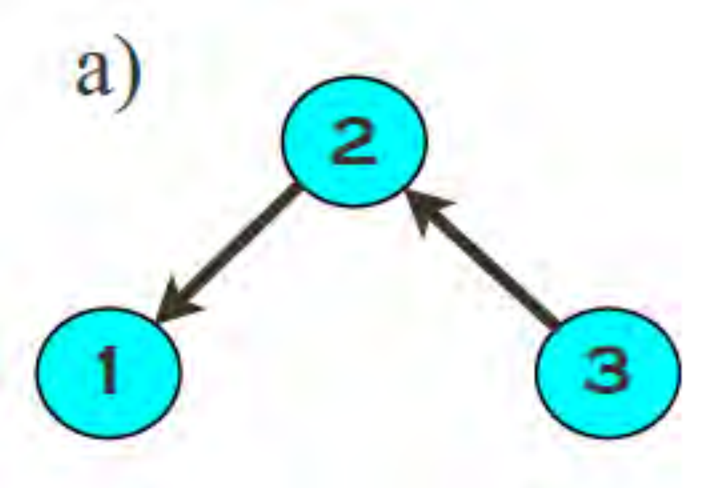
$$\ddot{x}_1 - \mu(1 - x_1^2)\dot{x}_1 + \omega_1^2 x_1 = \varepsilon[\sigma_{12}(x_2 + \dot{x}_2) + \sigma_{13}(x_3 + \dot{x}_3)] \,,$$
$$\ddot{x}_2 - \mu(1 - x_2^2)\dot{x}_2 + \omega_2^2 x_2 = \varepsilon[\sigma_{21}(x_1 + \dot{x}_1) + \sigma_{23}(x_3 + \dot{x}_3)] \,,$$
$$\ddot{x}_3 - \mu(1 - x_3^2)\dot{x}_3 + \omega_3^2 x_3 = \varepsilon[\sigma_{31}(x_1 + \dot{x}_1) + \sigma_{32}(x_2 + \dot{x}_2)] \,.$$

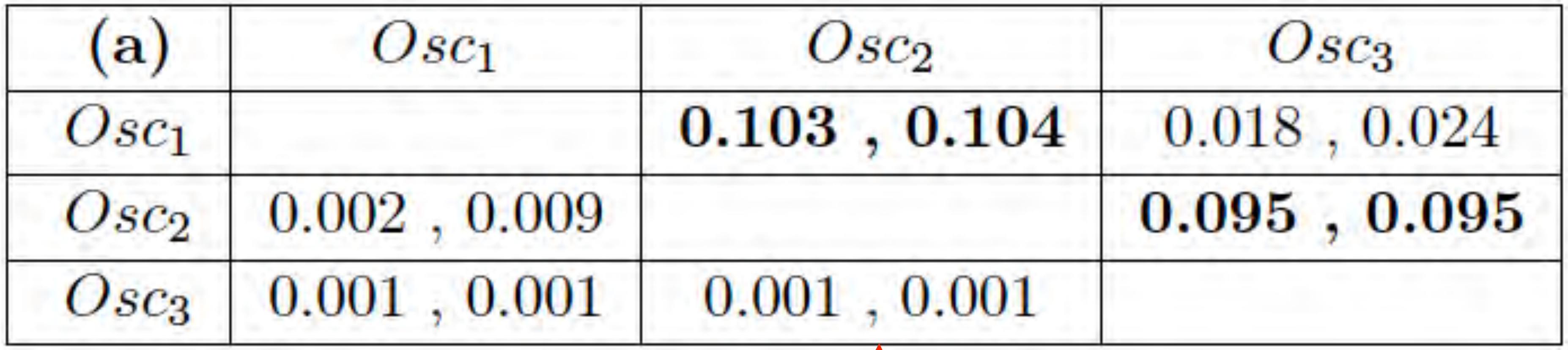
Parameters: $\varepsilon = 0.2$ $\mu = 0.5$

$\omega_1 = 1$ $\omega_2 = 1.3247$ $\omega_3 = 1.75483$

Connectivity matrix: $\sigma_{i,j} = 0$ or 1

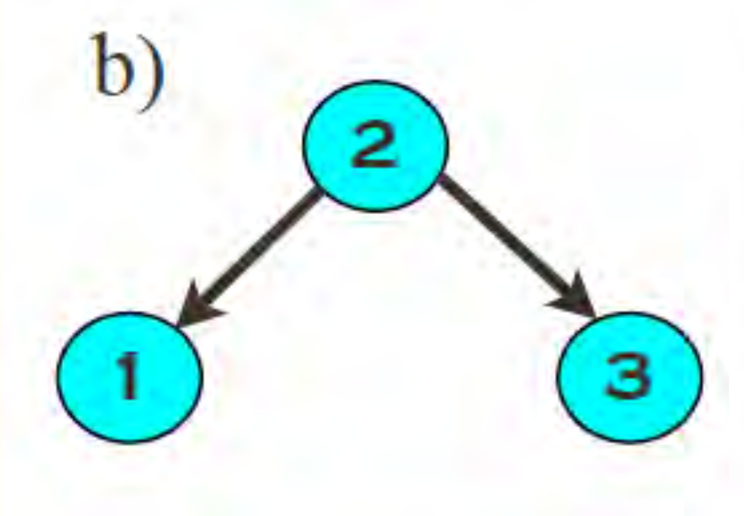
Example, *N*=3, results

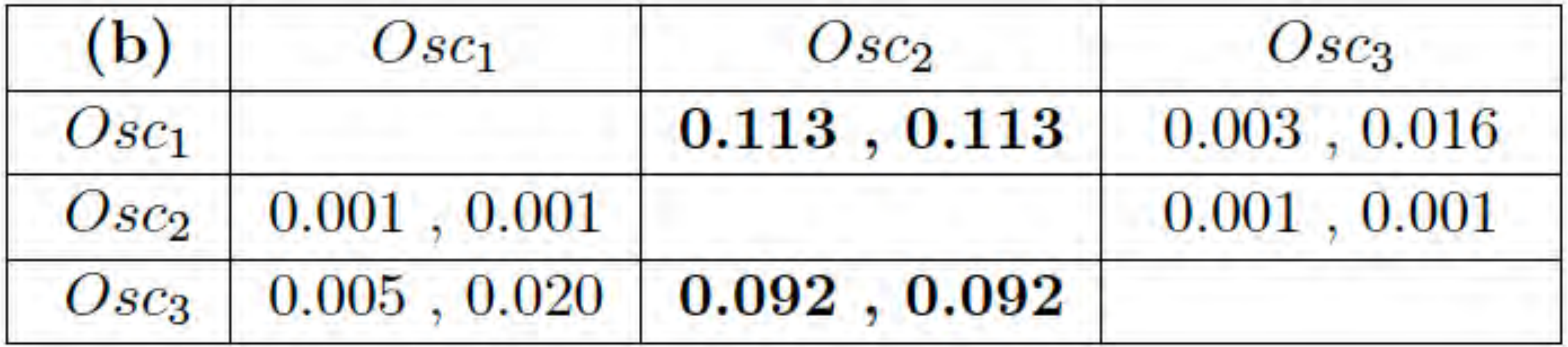




(a)	Osc_1	Osc_2	Osc_3
Osc_1		**0.103 , 0.104**	0.018 , 0.024
Osc_2	0.002 , 0.009		**0.095 , 0.095**
Osc_3	0.001 , 0.001	0.001 , 0.001	

$\mathcal{N}_{3\leftarrow2}, \mathcal{P}_{3\leftarrow2}$





(b)	Osc_1	Osc_2	Osc_3
Osc_1		**0.113 , 0.113**	0.003 , 0.016
Osc_2	0.001 , 0.001		0.001 , 0.001
Osc_3	0.005 , 0.020	**0.092 , 0.092**	

$\mathcal{N}_{3\leftarrow1}, \mathcal{P}_{3\leftarrow1}$

Remark: here $\mathcal{N}_{3\leftarrow2} = \mathcal{T}_{3\leftarrow2}$

Random oscillator network, *N*=5

$$\ddot{x}_k - \mu(1 - x_k^2)\dot{x}_k + \omega_k^2 x_k = \varepsilon \sum_l \sigma_{kl}(x_l \cos\Theta_{kl} + \dot{x}_l \sin\Theta_{kl})$$

σ_{kl}: random asymmetric connection matrix of zeros and ones

Fixed number of incoming connections (two)

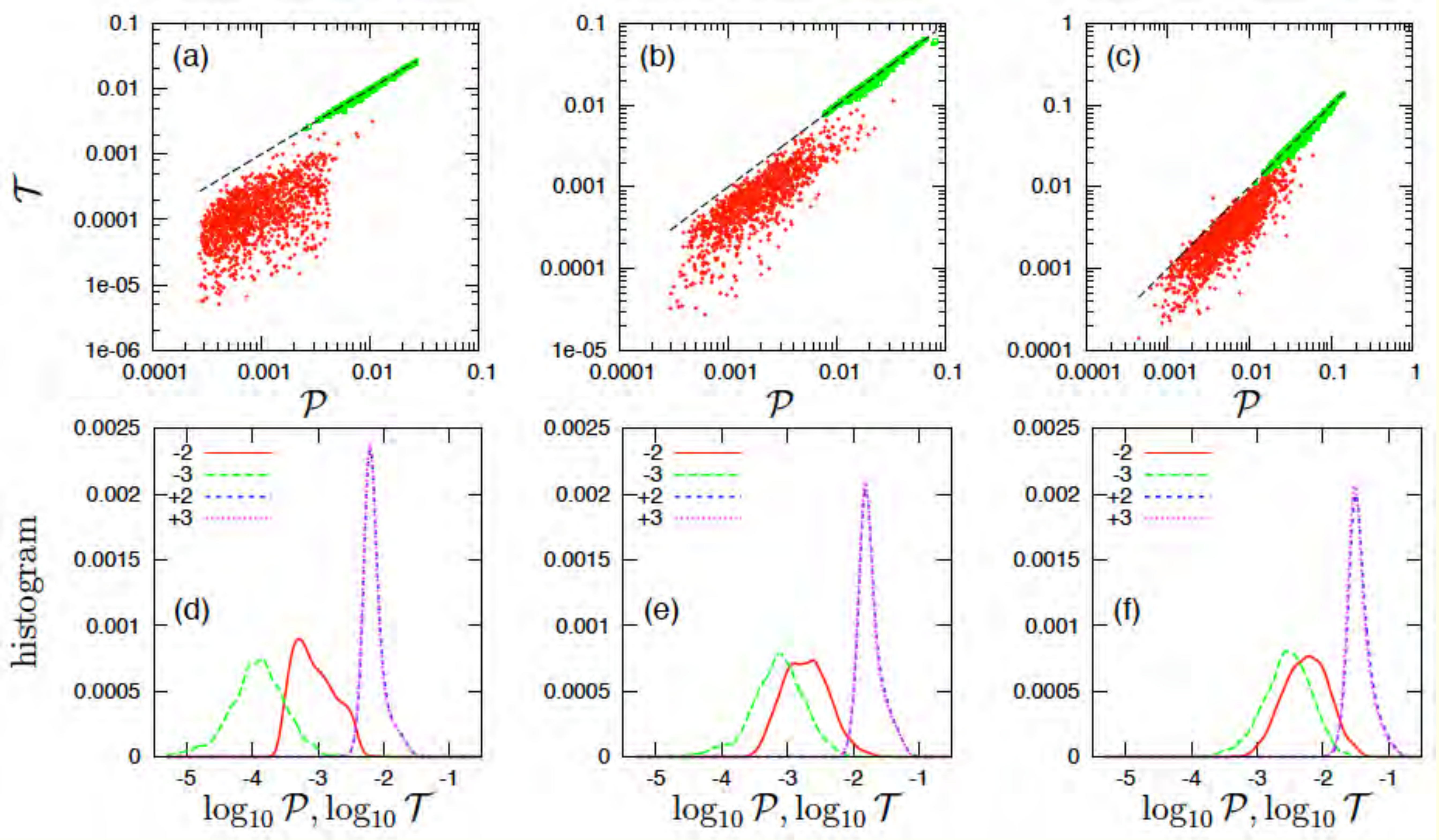
Frequencies are taken from a uniform distribution between 0.5 and 1.5

Θ_{kl} are taken from a uniform distribution between 0 and 2π

States with high degree of synchrony are excluded

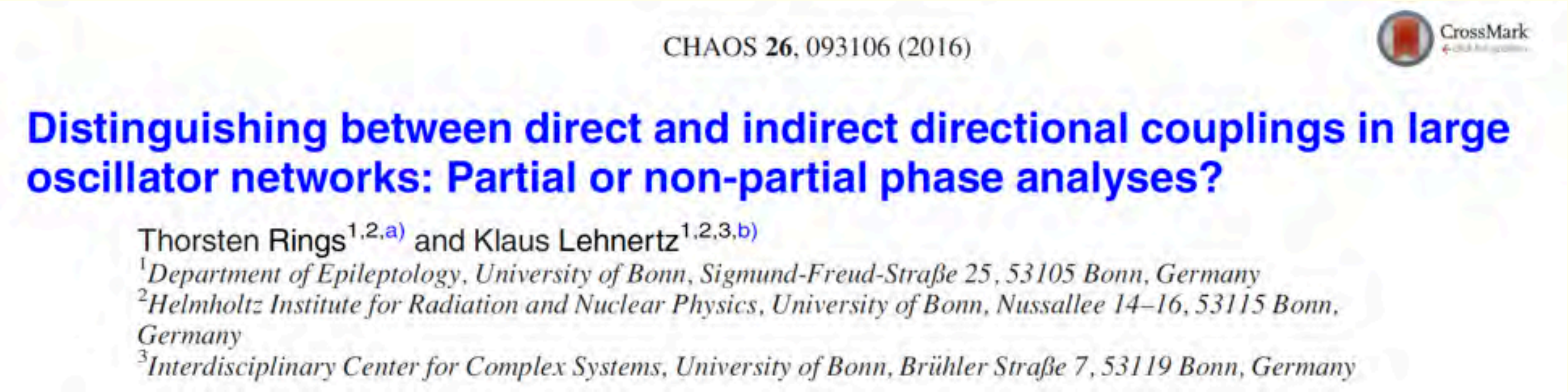
Random oscillator network, *N*=5, results

Existing connections in **green**, non-existing connections in **red**



$\varepsilon = 0.02$ $\varepsilon = 0.05$ $\varepsilon = 0.1$

Larger networks?



CHAOS **26**, 093106 (2016)

CrossMark

Distinguishing between direct and indirect directional couplings in large oscillator networks: Partial or non-partial phase analyses?

Thorsten Rings[1,2,a)] and Klaus Lehnertz[1,2,3,b)]
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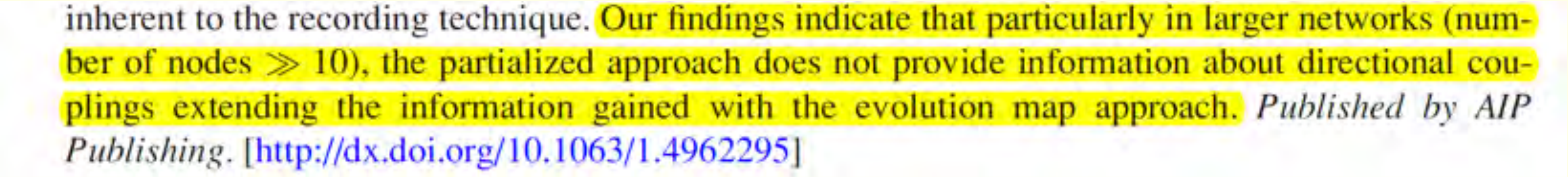
[2]*Helmholtz Institute for Radiation and Nuclear Physics, University of Bonn, Nussallee 14–16, 53115 Bonn, Germany*
[3]*Interdisciplinary Center for Complex Systems, University of Bonn, Brühler Straße 7, 53119 Bonn, Germany*



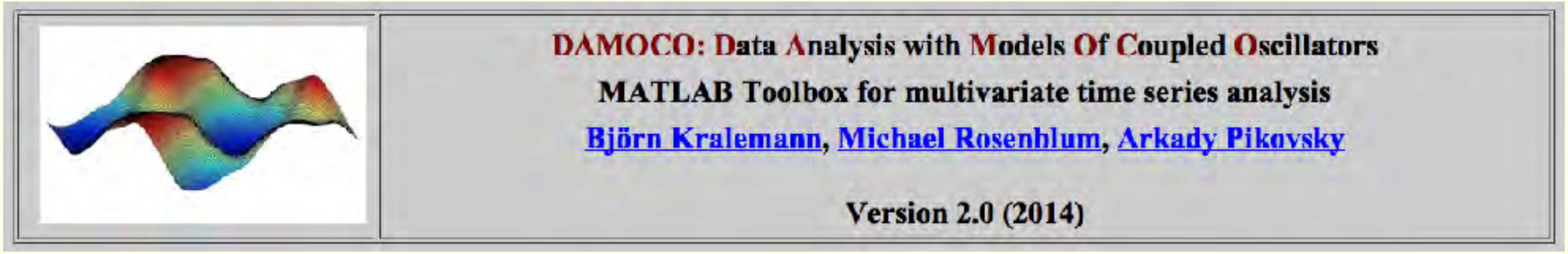
inherent to the recording technique. Our findings indicate that particularly in larger networks (number of nodes $\gg 10$), the partialized approach does not provide information about directional couplings extending the information gained with the evolution map approach. *Published by AIP Publishing.* [http://dx.doi.org/10.1063/1.4962295]

Conclusions

- Invariant reconstruction of phase equations for a network
- Characterization of directional couplings via partial norms
- Triplet analysis yields directed connectivity
- We detect effective phase connectivity, which is close but not equivalent to the structural connectivity

References

- B. Kralemann, A. Pikovsky, and M. Rosenblum, Chaos, 21, p. 025104, 2011
- B. Kralemann, A. Pikovsky, and M. Rosenblum, New J Phys, 16 085013, 2014



DAMOCO: Data Analysis with Models Of Coupled Oscillators

MATLAB Toolbox for multivariate time series analysis

Björn Kralemann, Michael Rosenblum, Arkady Pikovsky

Version 2.0 (2014)

Software for data analysis can be downloaded from:
www.stat.physik.uni-potsdam.de/~mros/damoco2.html

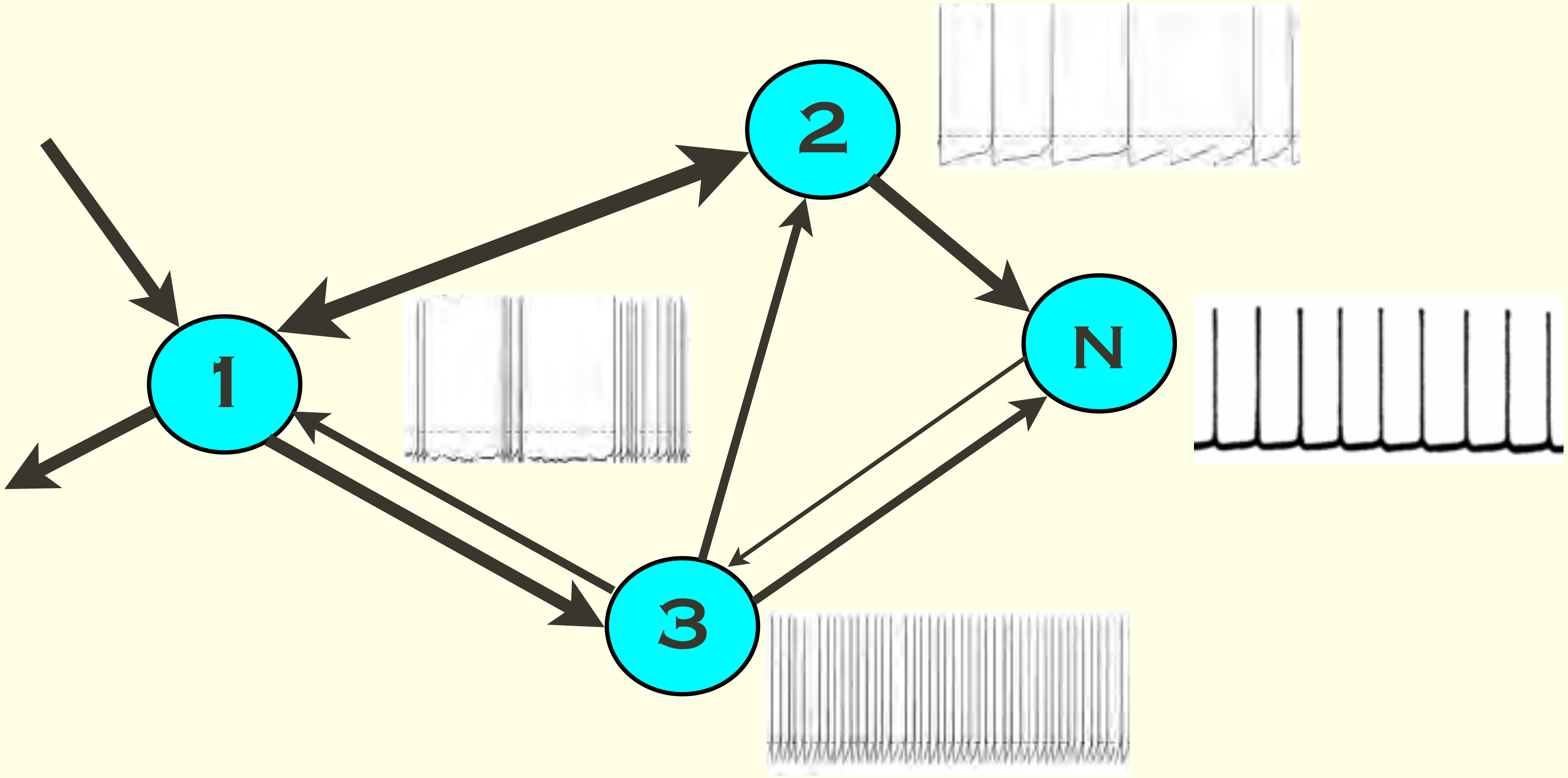
Reconstructing networks of pulse-coupled oscillators from spike trains

Rok Cestnik

COSMOS

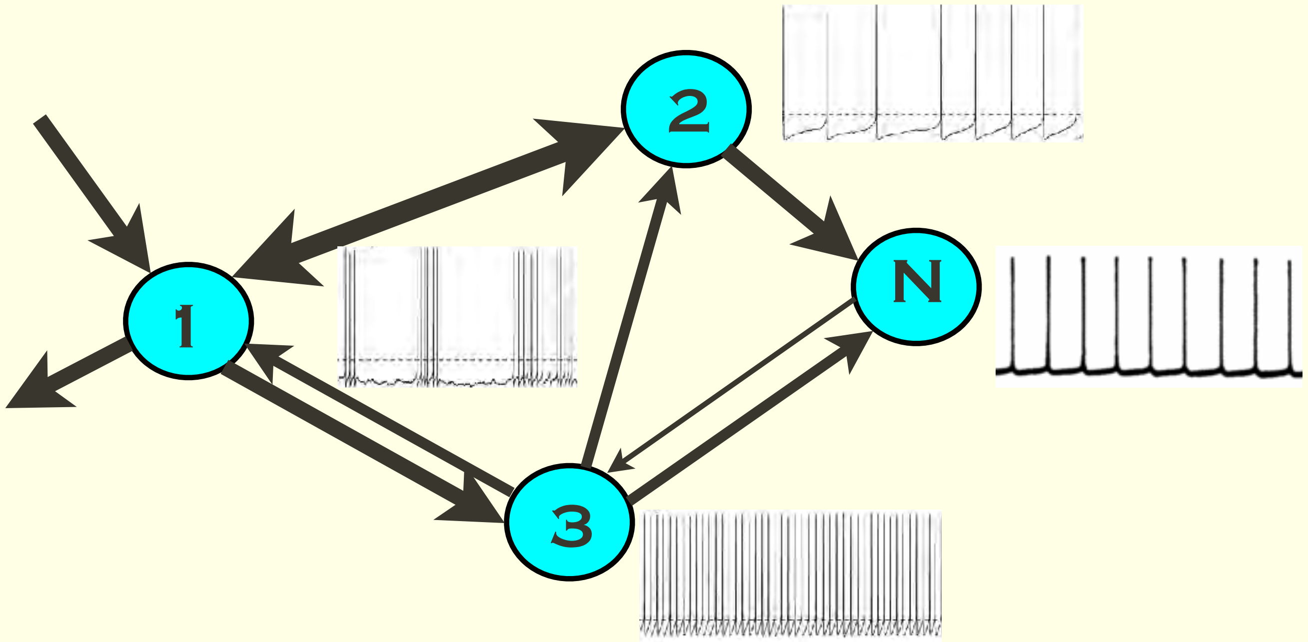
Complex Oscillatory Systems:
Modeling and Analysis
Innovative Training Network
European Joint Doctorate

Formulation of the problem



The data we measure are like **sequences of spikes**

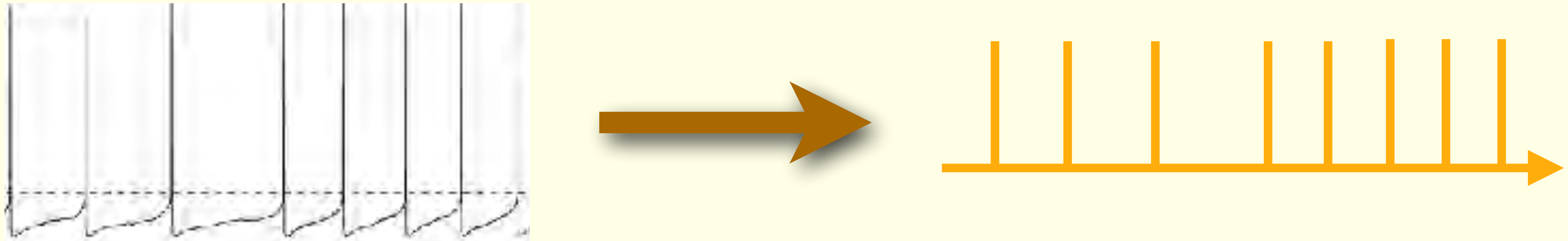
Formulation of the problem II



The data we measure are like **sequences of spikes**

→ we can reliably detect only times of spikes

→ we reduce the data to **point processes**



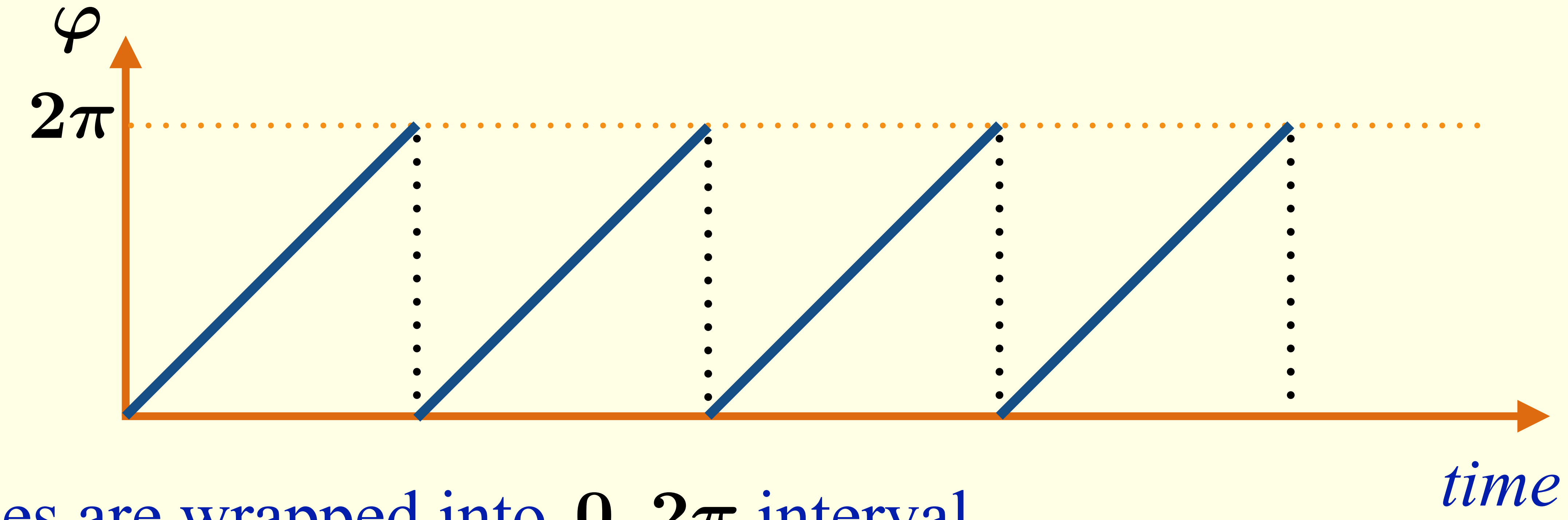
Assumptions about the network

- Weak interaction: phase description is justified
- PRC of a unit is the same for all incoming connections
  PRCs of different units can differ!
- Coupling is bidirectional but generally asymmetric,
  $\varepsilon_{km} \neq \varepsilon_{mk}$

strength of the link from m to k

A simple model: integrate-and-fire units

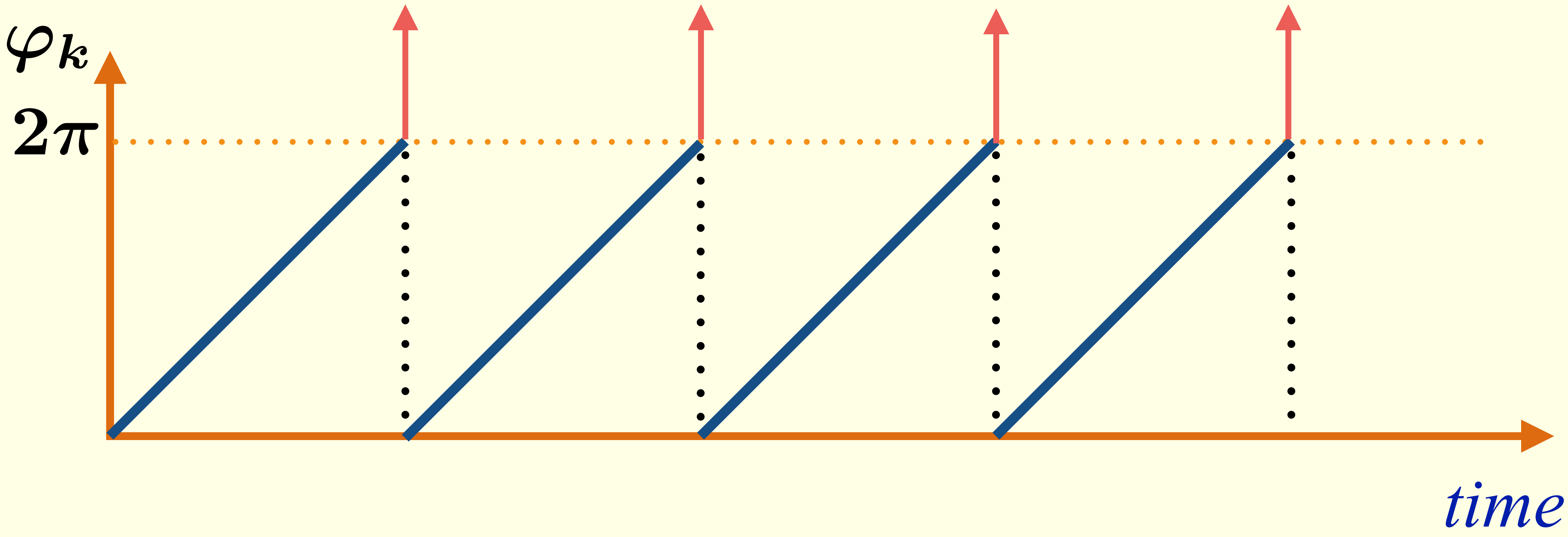
- Without interaction phases of all oscillators grow as $\varphi_k = \omega_k t$



phases are wrapped into $0, 2\pi$ interval

A simple model: integrate-and-fire units

- Without interaction phases of all oscillators grow as $\varphi_k = \omega_k t$
- When phase of the oscillator ***k*** attains $\varphi_k = 2\pi$, it **issues a spike**



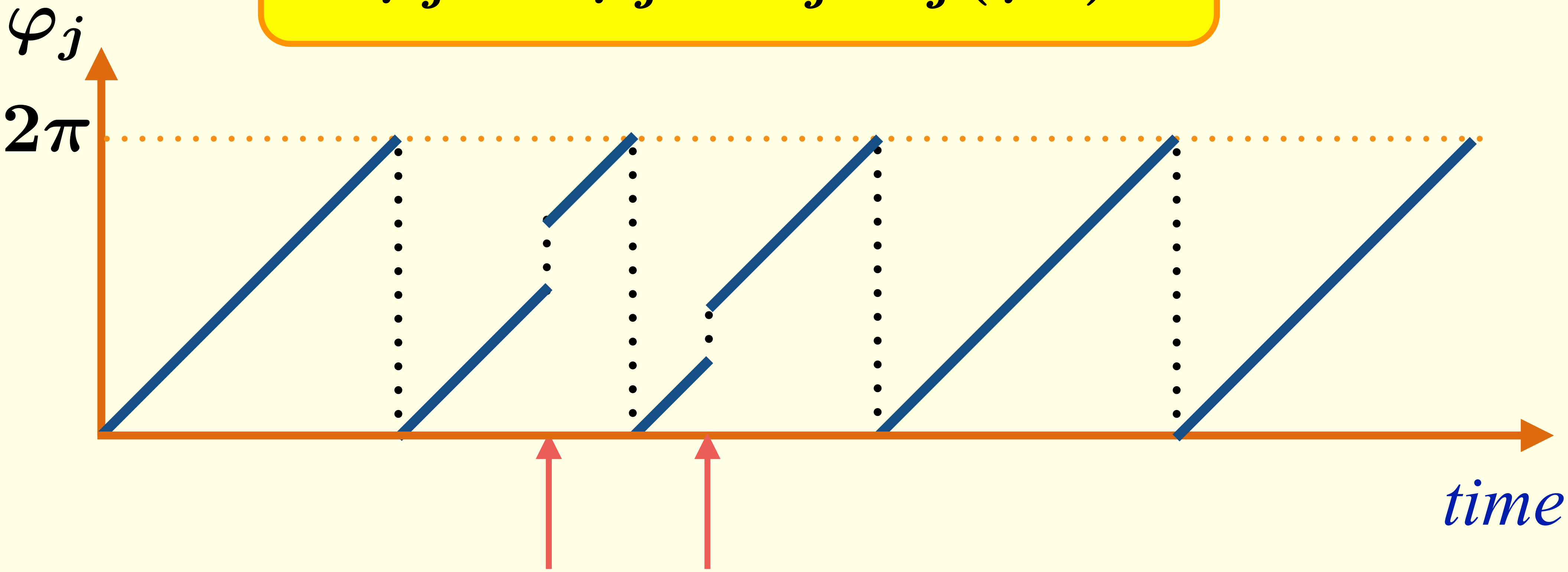
spikes affect all units with incoming connections from unit *k*

A simple model: integrate-and-fire units

- Without interaction phases of all oscillators grow as $\varphi_k = \omega_k t$
- When phase of the oscillator ***k*** attains $\varphi_k = 2\pi$, it **issues a spike**

- When unit *j* **receives** a spike from unit *k*, its phase is instantaneously reset according to its PRC $Z_j(\varphi)$:

$$\boxed{\varphi_j \to \varphi_j + \varepsilon_{jk} Z_j(\varphi_k)}$$



Assumptions

- Weak interaction: phase description is justified
- PRC of a unit is the same for all incoming connections
  PRCs of different units can differ!
- Coupling is bidirectional but generally asymmetric,
  $\varepsilon_{km} \neq \varepsilon_{mk}$
- Relaxation after pulse stimulation is fast

Our approach: iterative solution

- We choose one oscillator (let it be the first one) and consider its all incoming connections ε_{1m}

- For this oscillator, we recover:
  - its frequency
  - its PRC
  - strength of all incoming connections

- We achieve this in several iterative steps

- Then we repeat the procedure for all other units

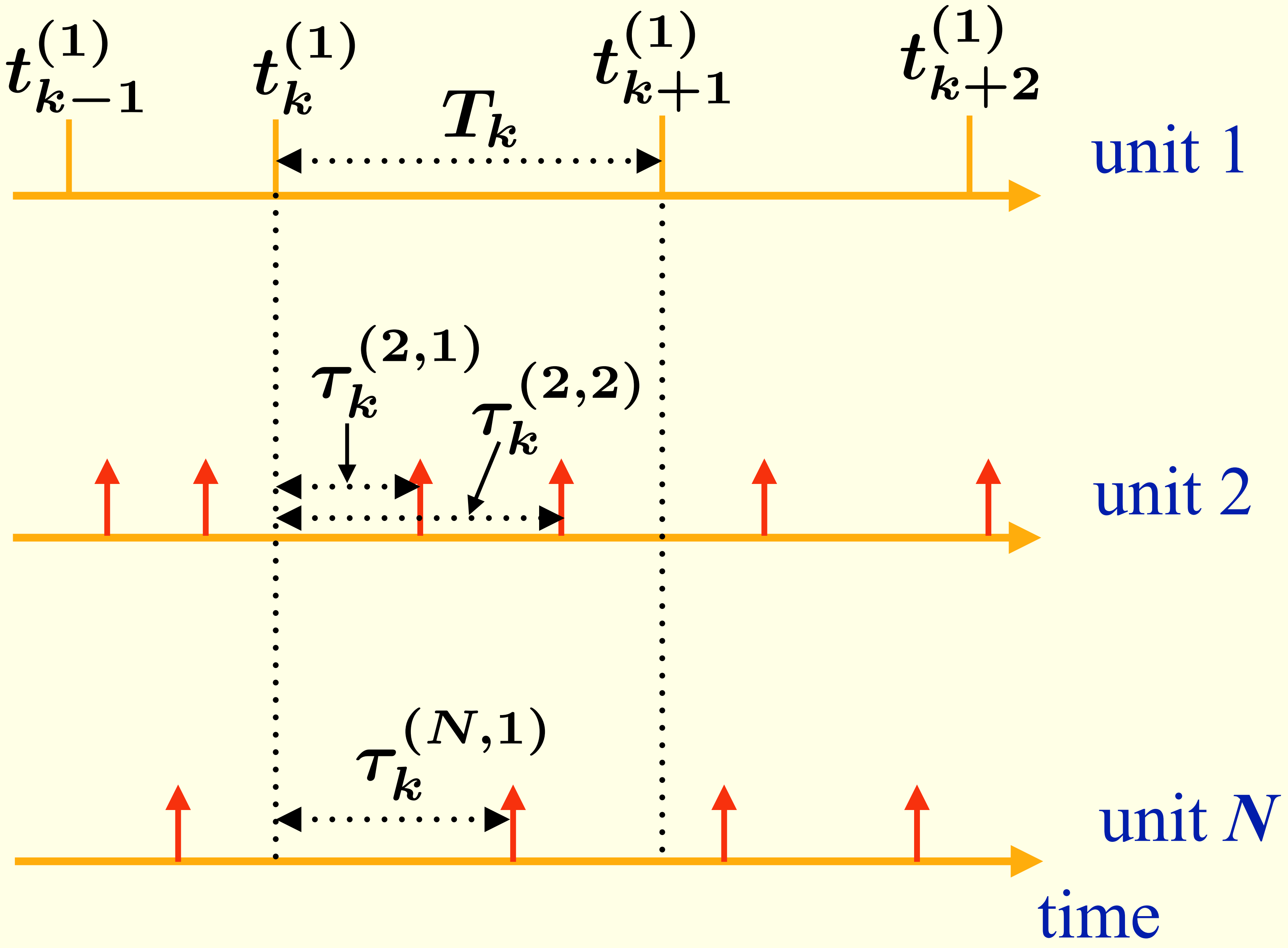
Our approach: Notations

- Since we choose the first oscillator, we simplify notations by omitting one index

- For this oscillator, we recover:

  - its frequency ω
  - its PRC $Z(\varphi)$
  - strength of all incoming connections $\varepsilon_m, m = 2, \ldots, N$

Notations II



When the spike at $\tau_k^{(i,l)}$ arrives, the phase of the first unit is

$$\varphi(t_k^{(1)} + \tau_k^{(i,l)}) = \varphi_k^{(i,l)}$$

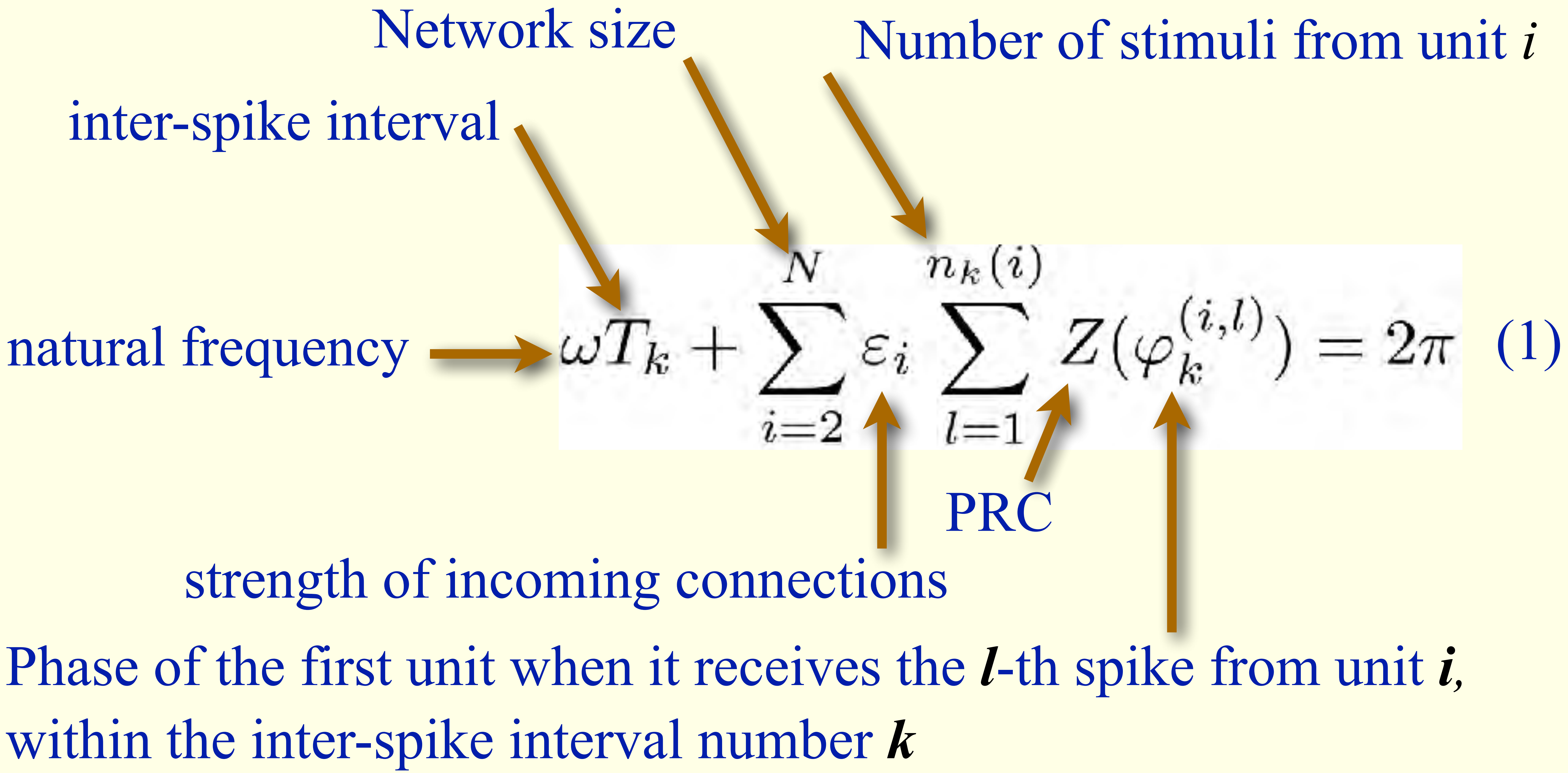
Phase equation

Phase increase within each inter-spike interval is **2π**

$$\omega T_k + \sum_{i=2}^{N} \varepsilon_i \sum_{l=1}^{n_k(i)} Z(\varphi_k^{(i,l)}) = 2\pi \qquad (1)$$

Phase equation

Phase increase within each inter-spike interval is **2π**



Network size

Number of stimuli from unit ***i***

inter-spike interval

natural frequency

$$\omega T_k + \sum_{i=2}^{N} \varepsilon_i \sum_{l=1}^{n_k(i)} Z(\varphi_k^{(i,l)}) = 2\pi \quad (1)$$

PRC

strength of incoming connections

Phase of the first unit when it receives the ***l***-th spike from unit ***i***, within the inter-spike interval number ***k***

Our approach: main idea

$$\omega T_k + \sum_{i=2}^{N} \varepsilon_i \sum_{l=1}^{n_k(i)} Z(\varphi_k^{(i,l)}) = 2\pi \qquad (1)$$

- Suppose we know phases and coupling coefficients; then we represent the PRC as a finite Fourier series; thus, we obtain M linear equations (1), where M is the number of inter-spike intervals; for long time series it can be solved, e.g., by LMS fit

- Suppose, vice versa, that we know phases and PRC; then we obtain a linear system to find coupling coefficients ε_j

Our approach: main idea

$$\omega T_k + \sum_{i=2}^{N} \varepsilon_i \sum_{l=1}^{n_k(i)} Z(\varphi_k^{(i,l)}) = 2\pi \qquad (1)$$

Thus:

- φ_k, ε_i are known → we find Z, ω
- φ_k, Z is known → we find ε_i, ω

Our approach: iterative solution

Thus:

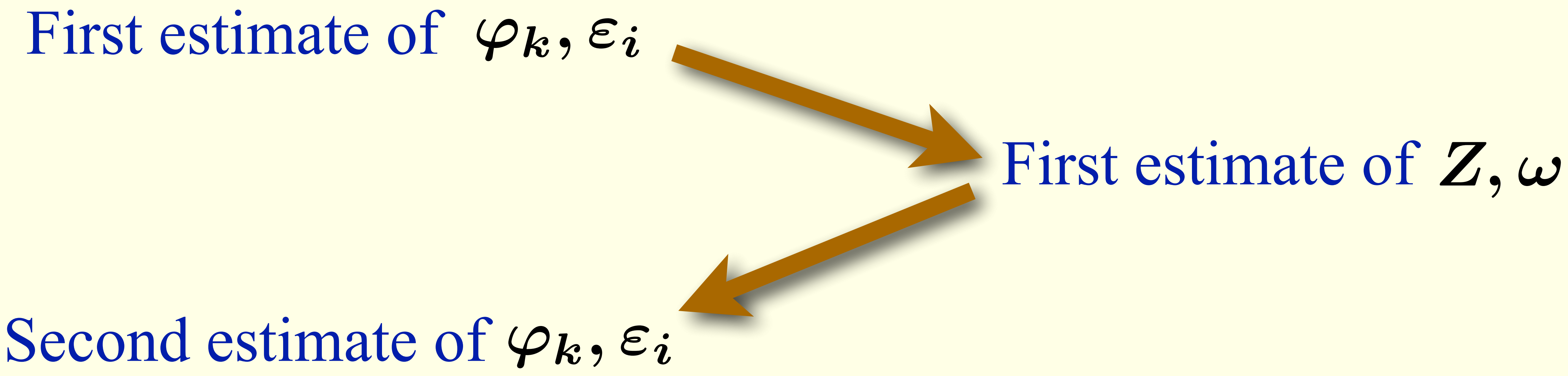
- φ_k, ε_i are known → we find Z, ω
- φ_k, Z is known → we find ε_i, ω

First estimate of φ_k, ε_i → First estimate of Z, ω

Our approach: iterative solution

Thus:

- φ_k, ε_i are known → we find Z
- φ_k, Z is known → we find ε_i



First estimate of φ_k, ε_i

First estimate of Z, ω

Second estimate of φ_k, ε_i

Our approach: iterative solution

Thus:

- $\boldsymbol{\varphi_k, \varepsilon_i}$ are known → we find Z
- $\boldsymbol{\varphi_k, Z}$ is known → we find $\boldsymbol{\varepsilon_i}$

First estimate of $\boldsymbol{\varphi_k, \varepsilon_i}$

First estimate of $\boldsymbol{Z, \omega}$

Second estimate of $\boldsymbol{\varphi_k, \varepsilon_i}$

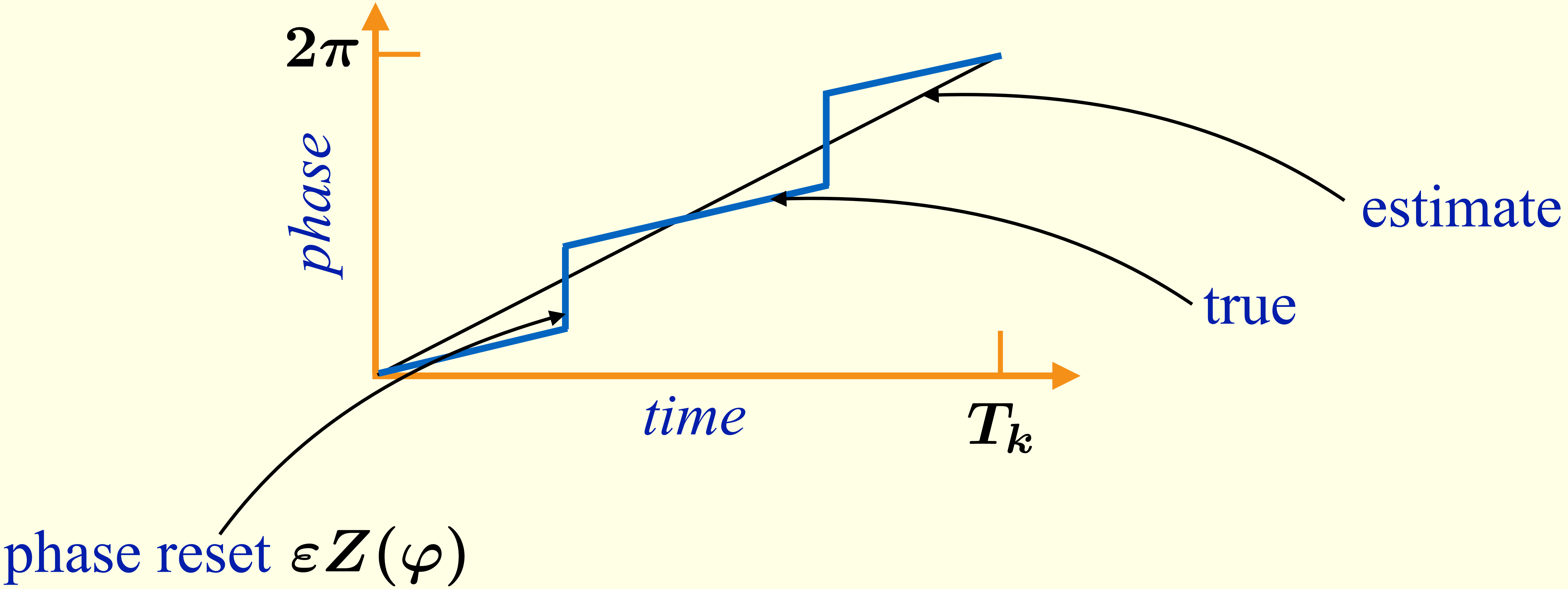
Second estimate of $\boldsymbol{Z, \omega}$

Third estimate of $\boldsymbol{\varphi_k, \varepsilon_i}$

$\dots$

First estimate: phases

Initial estimate: proportionally to time $\varphi_k^{(i,l)} = 2\pi\tau_k^{(i,l)}/T_k$

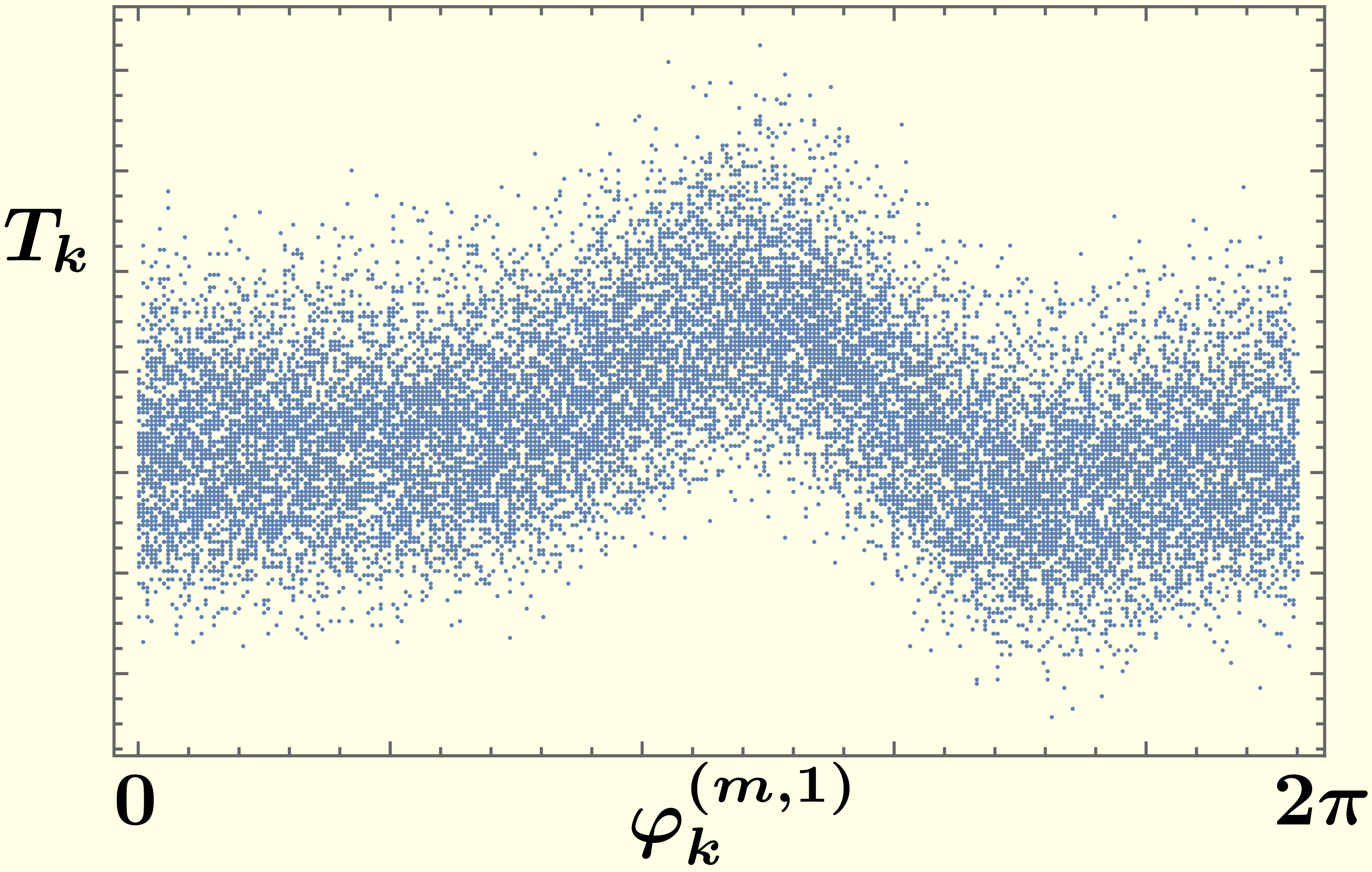


Error of the initial estimate is of the order of $\varepsilon Z(\varphi)$

First estimate: coupling coefficients

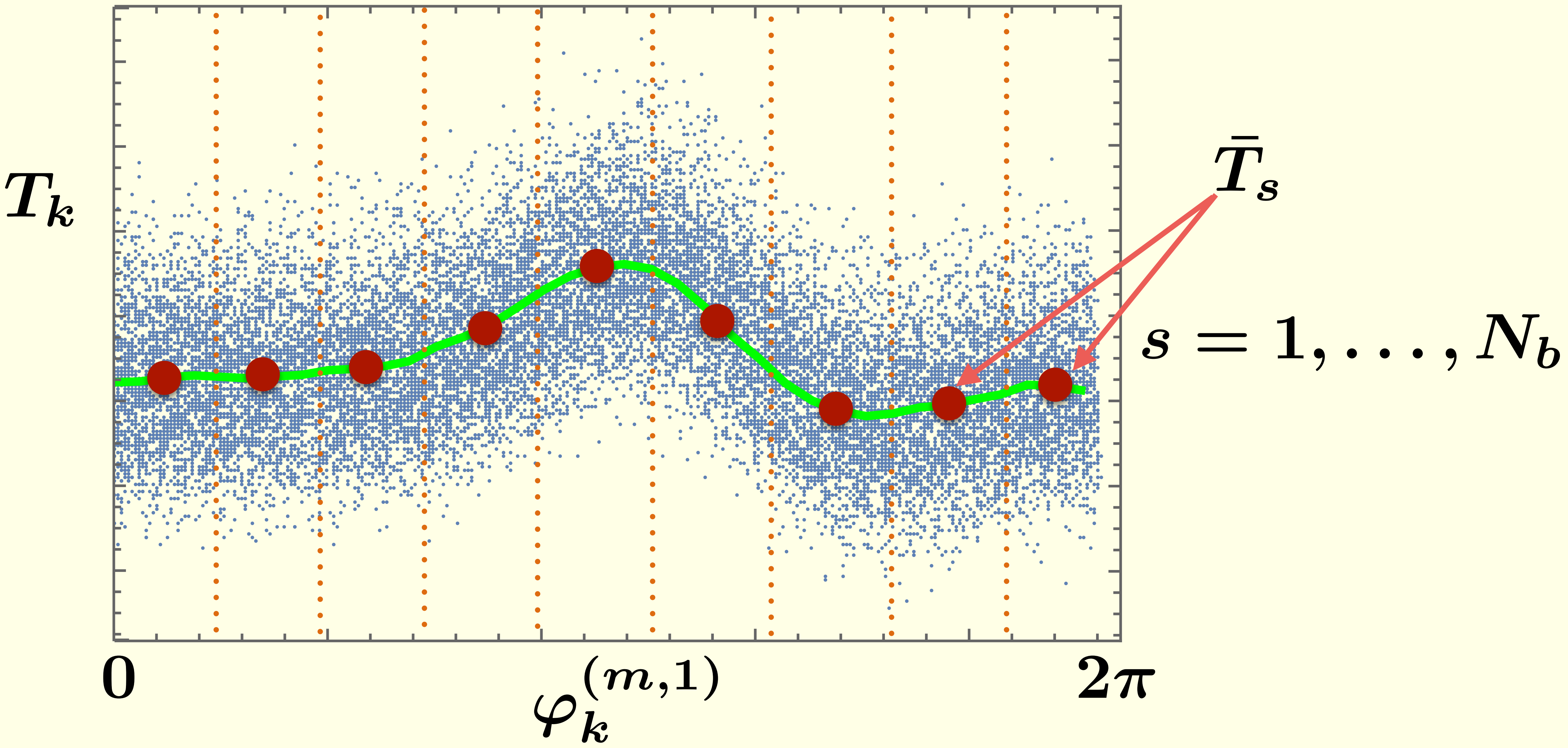
We want to estimate strength of the link from unit ***m*** to unit ***1***

We plot the interval length T_k of the first unit vs the phase when the first stimulus from unit ***m*** arrives within this time interval



First estimate: coupling coefficients

Binning and averaging over N_b bins

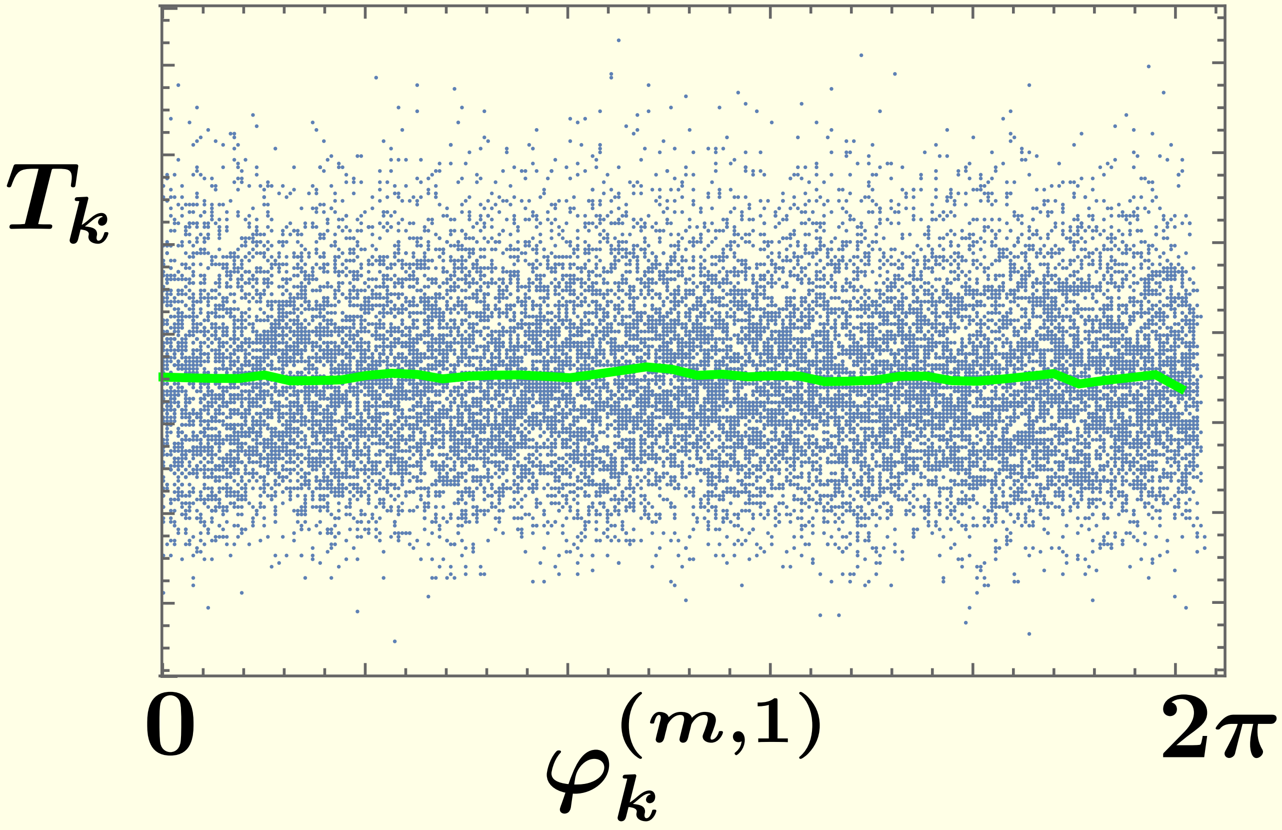


First estimate: $\varepsilon_m = \left\langle \left(\bar{T}_s - \langle \bar{T}_s \rangle \right)^2 \right\rangle^{1/2}$

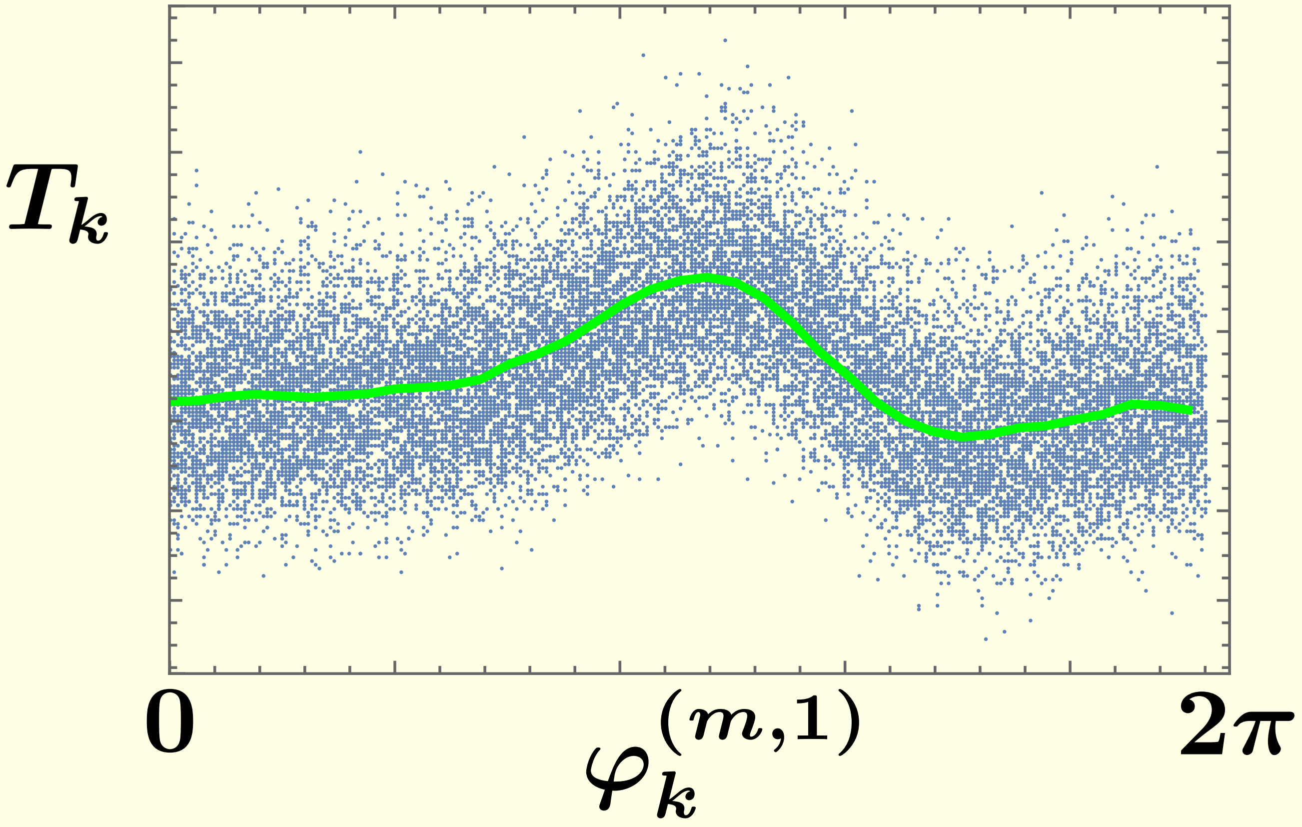
averaging over s

First estimate: coupling coefficients

Very weak coupling:
no dependence



Stronger coupling:
prominent dependence



First estimate: $\varepsilon_m = \langle \big(\bar{T}_s - \langle \bar{T}_s \rangle\big)^2 \rangle^{1/2}$

Next estimates: phases

An example: within T_k there are three incoming stimuli at

$$\tau_k^{(i,1)} < \tau_k^{(m,1)} < \tau_k^{(n,1)}$$

1st stimulus: $\varphi_k^{(i,1)} = \omega\tau_k^{(i,1)}$

2nd stimulus: $\varphi_k^{(m,1)} = \omega\tau_k^{(m,1)} + \varepsilon_i Z(\varphi_k^{(i,1)})$

3rd stimulus: $\varphi_k^{(n,1)} = \omega\tau_k^{(n,1)} + \varepsilon_i Z(\varphi_k^{(i,1)}) + \varepsilon_m Z(\varphi_k^{(m,1)})$

At the end of the interval:

$$\psi = \omega T_k + \varepsilon_i Z(\varphi_k^{(i,1)}) + \varepsilon_m Z(\varphi_k^{(m,1)}) + \varepsilon_n Z(\varphi_k^{(n,1)})$$

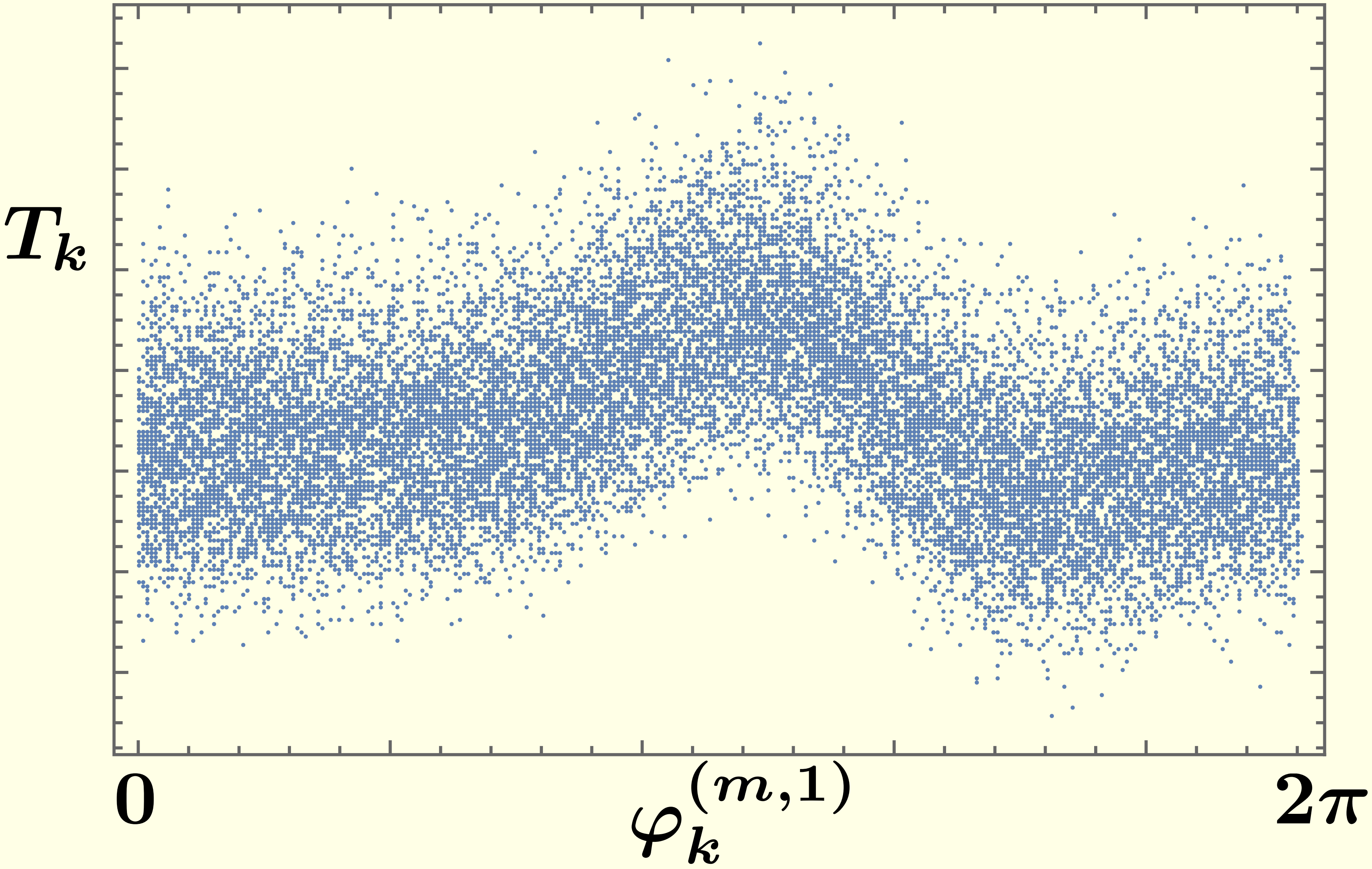
Our quantities are not precise → generally $\psi \neq 2\pi$

→ we rescale all estimated phases by $2\pi/\psi$

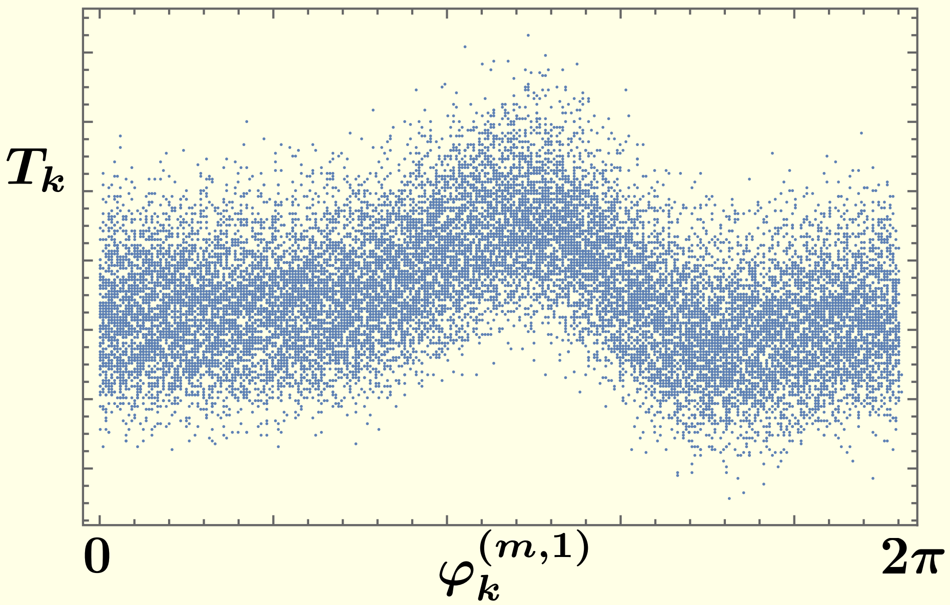
Coupling coefficients once again

We want to estimate strength of the link from unit ***m*** to unit ***1***

We plot the interval length T_k of the first unit vs the phase when the first stimulus from unit ***m*** arrives within this time interval



Coupling coefficients once again



This approach works very good for a rather long time series

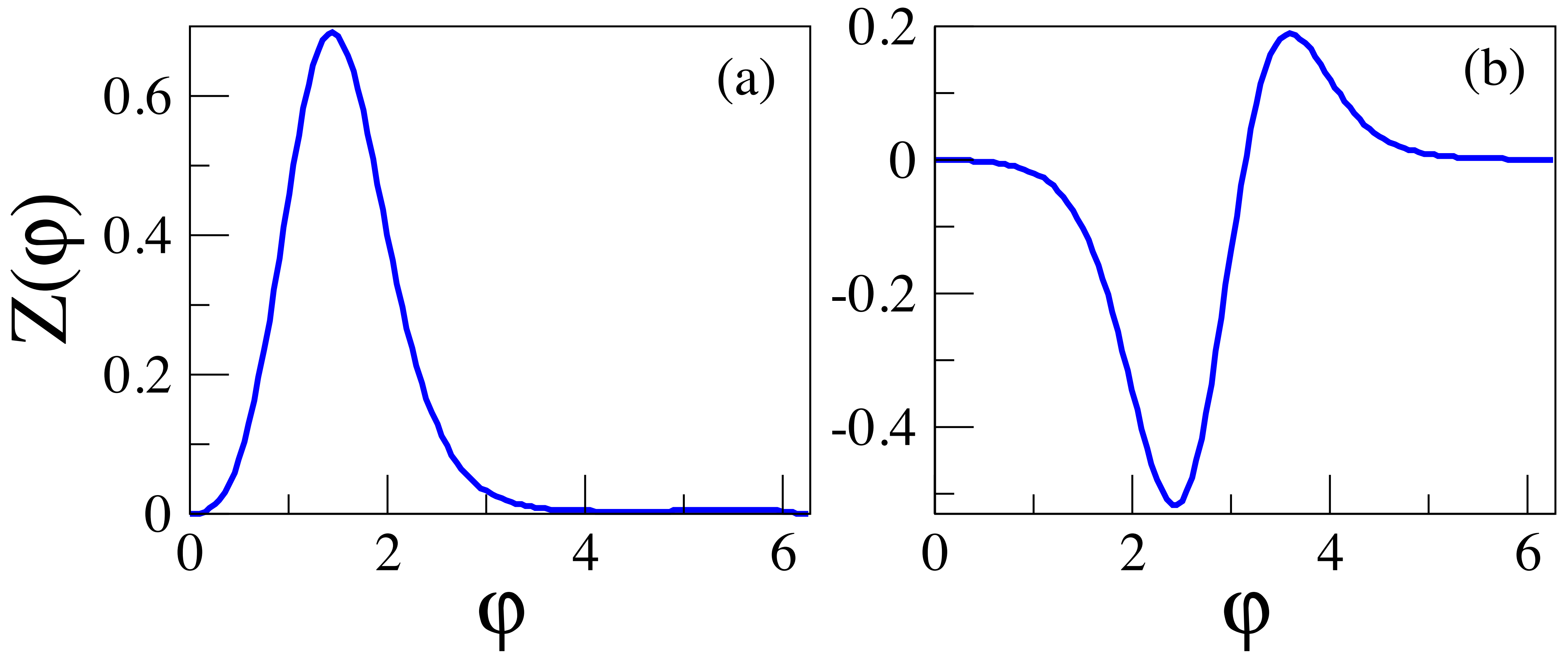
Numerical tests demonstrate that iterations converge to the correct value even for random assignment of initial values ε_i !

Numerical tests

Model phase response curves

Type I PRC

Type II PRC



Numerical tests: a remark on normalization

Recall the main equation:

$$\omega T_k + \sum_{i=2}^{N} \varepsilon_i \sum_{l=1}^{n_k(i)} Z(\varphi_k^{(i,l)}) = 2\pi$$

ε_i *and* Z enter it as a product

→ ε_i *and* Z can be arbitrary rescaled

For comparison with the true values we choose scaling factor by minimizing

$$\sum_{i=2}^{N} \left[\varepsilon_i^{(t)} - c\varepsilon_i^{(r)}\right]$$

Numerical tests: a remark on normalization

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true

recovered

Numerical test I

Network size: $N = 20$

Natural frequencies: uniformly distributed between 1 and 2

$\omega_1 = 1$ (most difficult case)

Coupling coefficients: sampled from the positive part of a Gaussian distribution with zero mean and std 0.02

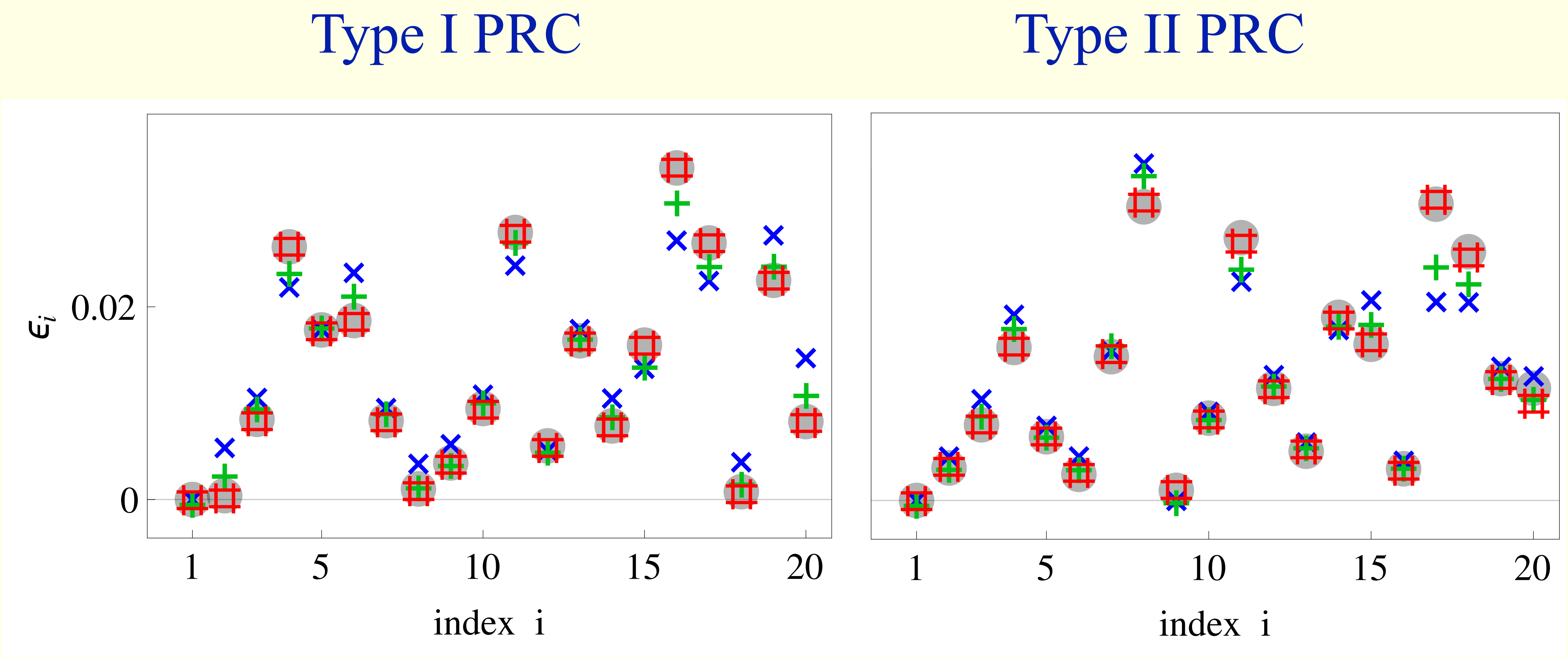
We exclude the networks where at least two units synchronize!

Reconstruction: 10 iterations, 10 Fourier harmonics

only 200 inter-spike intervals used

initial values $\varepsilon_i = 1, \ \forall i$

Iterative solution: results, coupling strength

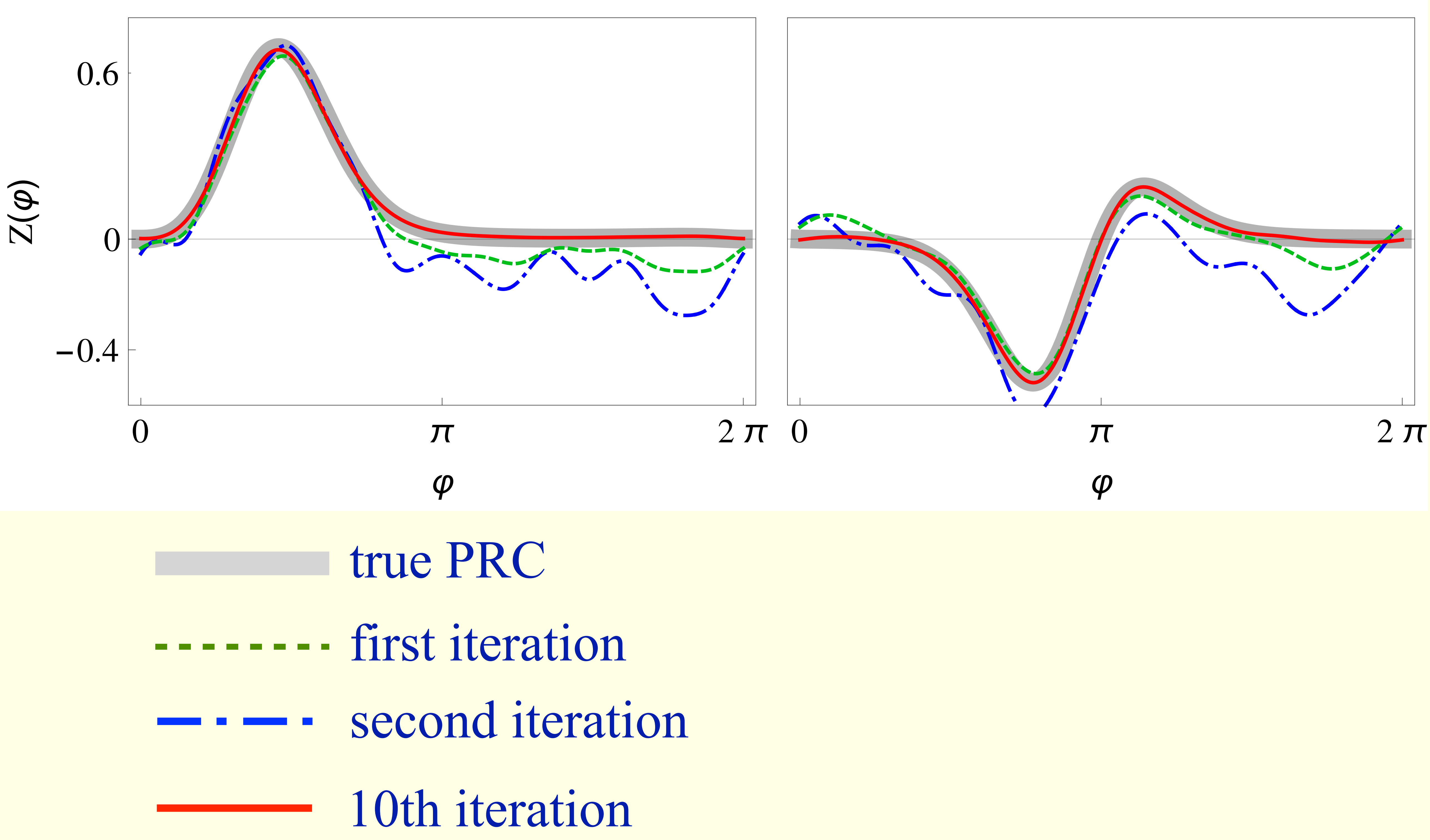


- ● true values
- \+ first iteration
- × second iteration
- ⌗ 10th iteration

Iterative solution: results, PRC

Type I PRC

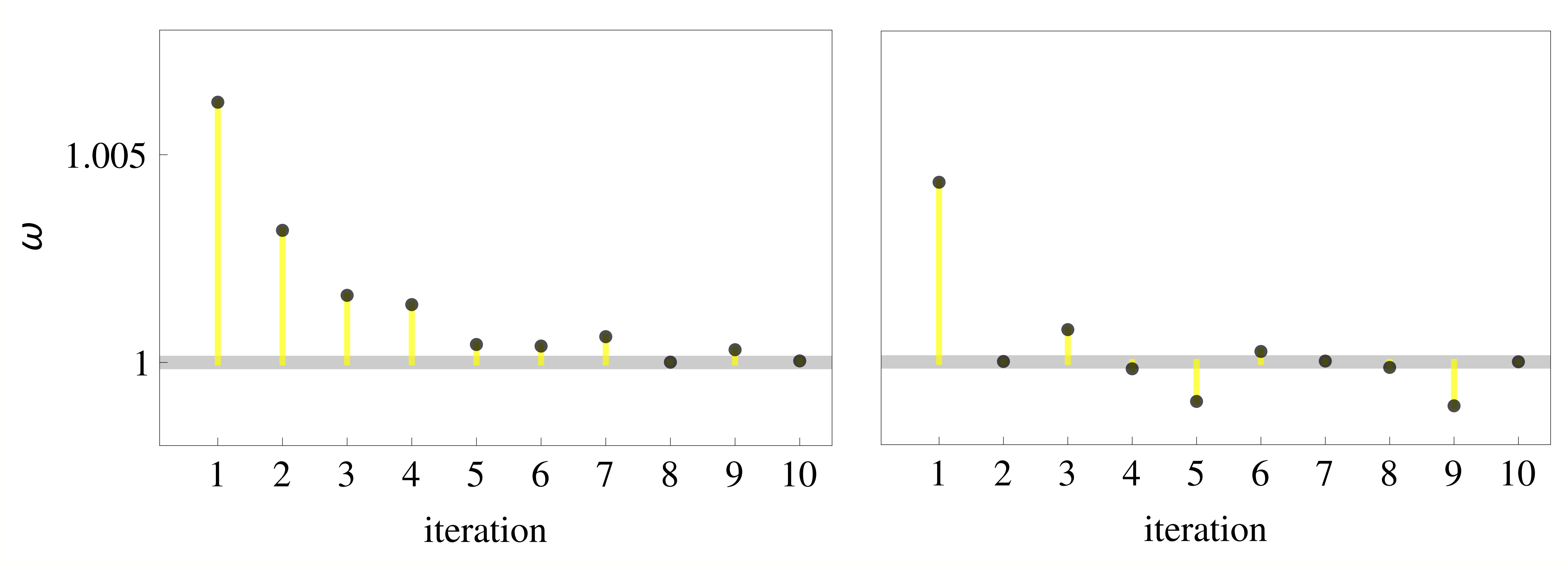
Type II PRC



Iterative solution: results, frequencies

Type I PRC

Type II PRC



▬ true value

Numerical test II: statistical analysis

Network size: $N = 20$

Natural frequencies: uniformly distributed between 1 and 2

$\omega_1 = 1$ (most difficult case)

Coupling coefficients: sampled from the positive part of a Gaussian distribution with zero mean and std 0.02

We exclude the networks where at least two units synchronize!

Reconstruction: 10 iterations, 10 Fourier harmonics

only 200 inter-spike intervals used

initial values $\varepsilon_i = 1, \ \forall i$

We generate and reconstruct 10^5 networks

Numerical test II: statistical analysis

Quality of the reconstruction: we define the corresponding errors

$$\Delta^2_{\mathrm{PRC}} = \frac{\int_0^{2\pi} \left[Z^{(\mathrm{t})}(\varphi) - Z^{(\mathrm{r})}(\varphi)\right]^2 d\varphi}{\int_0^{2\pi} \left[Z^{(\mathrm{t})}(\varphi)\right]^2 d\varphi} ,$$

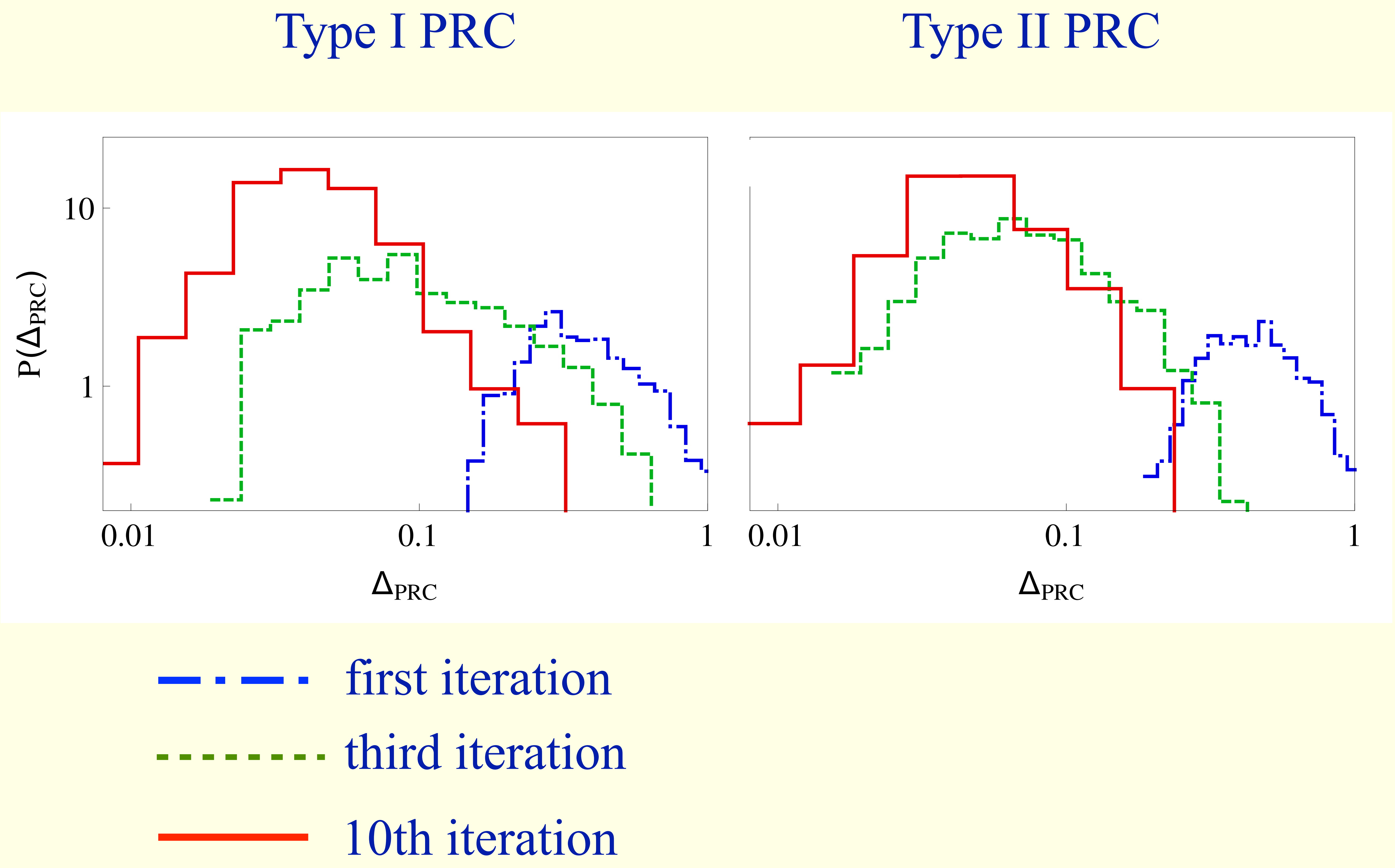
$$\Delta^2_{\varepsilon} = \sum_{i=2}^{N} \left[\varepsilon_i^{(\mathrm{t})} - \varepsilon_i^{(\mathrm{r})}\right]^2 \Big/ \sum_{i=2}^{N} \left[\varepsilon_i^{(\mathrm{t})}\right]^2 ,$$

$$\Delta^2_{\omega} = \left[\omega^{(\mathrm{t})} - \omega^{(\mathrm{r})}\right]^2$$

true

recovered

Numerical test II: results, histograms of errors



Type I PRC

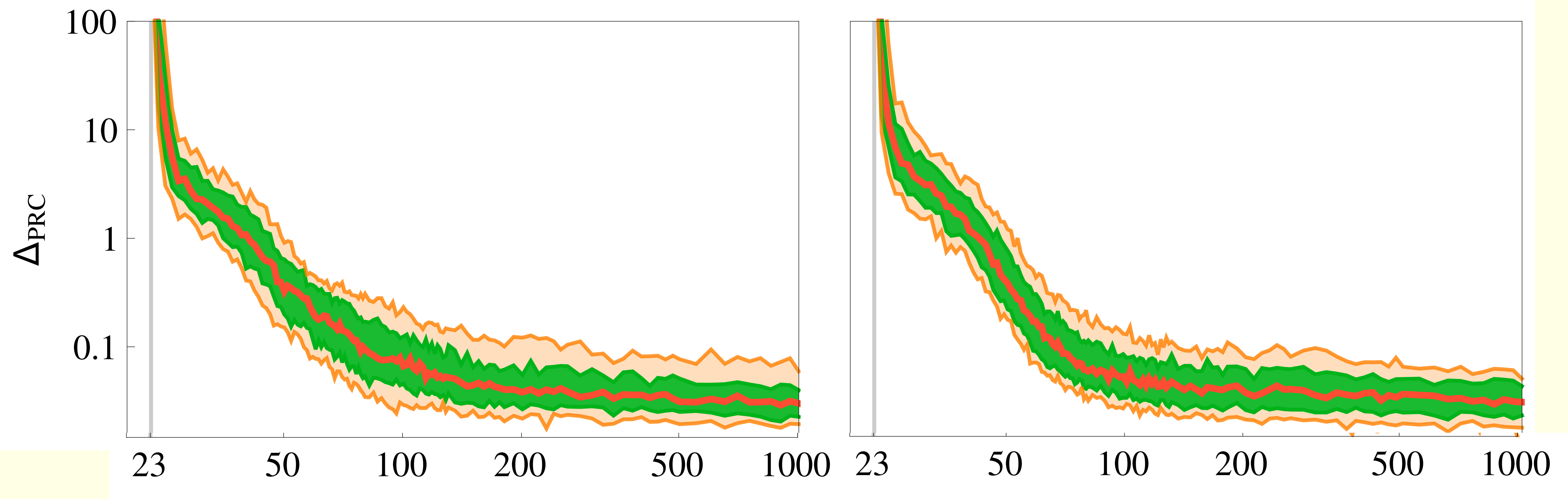
Type II PRC

first iteration

third iteration

10th iteration

Numerical test II: results, impact of data length



number of inter-spike intervals used

Further tests: impact of network size and noise

Network size from ***N=10*** to ***500***, with number of spikes ~N

Computational time: ***~N^4,*** in fact, small (minutes on a laptop)

Errors increase linearly with noise intensity

One step towards realistic modelling: Morris-Lecar neurons

$$\begin{aligned}\dot{V}_i =& I_i - g_l(V_i - V_l) - g_K w_i (V_i - V_k) \\ & - g_{Ca} m_\infty(V_i)(V_{Ca} - V_i) + I_i^{(\mathrm{syn})} \ , \\ \dot{w}_i =& \lambda(V_i)(w_\infty(V_i) - w_i) \ ,\end{aligned}$$

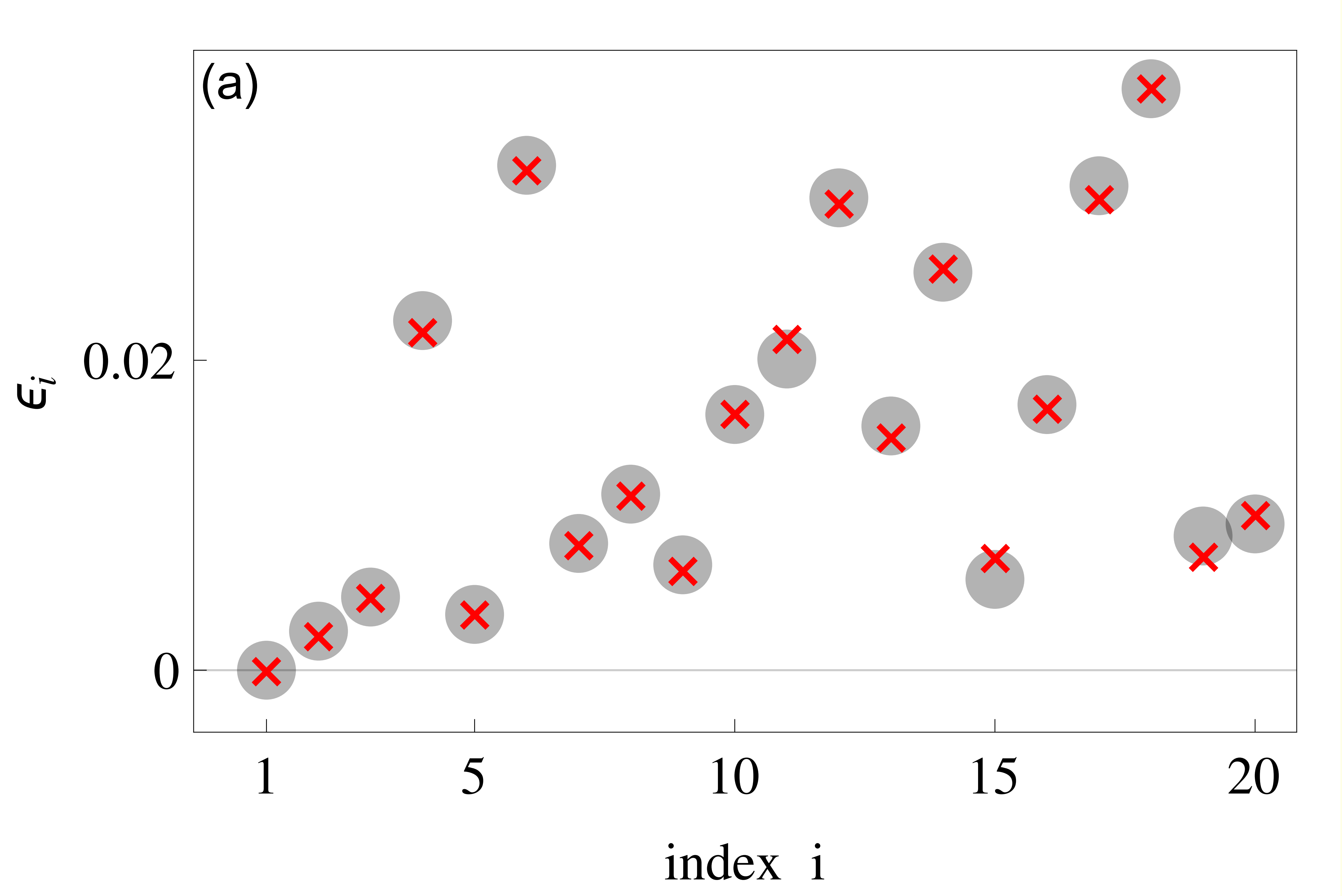
$$m_\infty(V) = [1 + \tanh\left(V - V_1/V_2\right)]/2 \ ,$$

$$w_\infty(V) = [1 + \tanh\left(V - V_3/V_4\right)]/2 \ ,$$

$$\lambda(V) = \cosh\left[(V - V_3)/(2V_4)\right]/3 \ ,$$

with synaptic coupling $I_i^{(\mathrm{syn})} = [V_{\mathrm{rev}} - V_i] \sum_{k, k \neq i} \frac{\varepsilon_{ik}}{1 + \exp\left[-(V_{\boldsymbol{k}} - V_{\mathrm{th}})/\sigma\right]}$

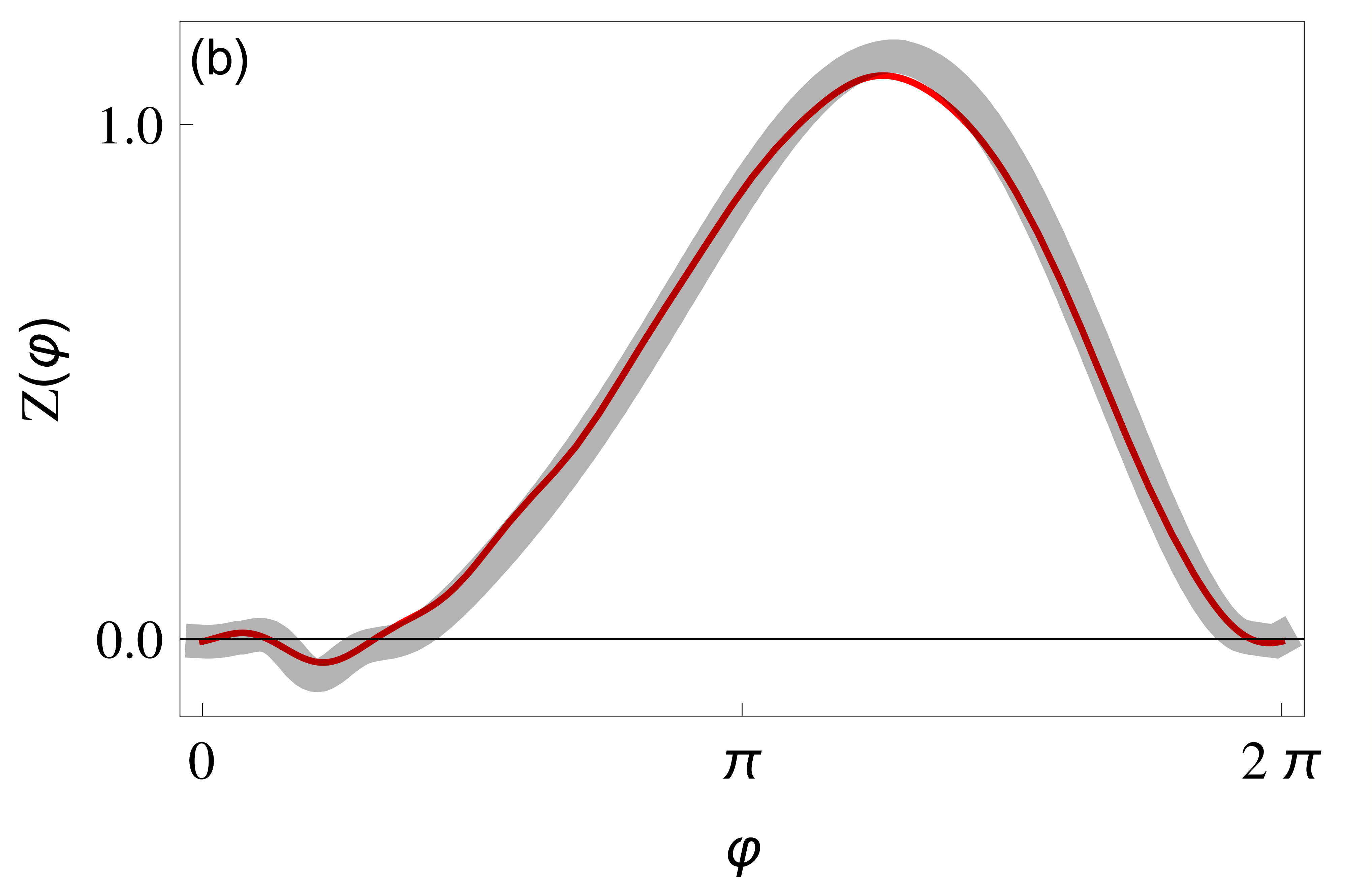
Morris-Lecar network: results, coupling strength



● true value ✕ after 10 iterations

only 200 inter-spike intervals are used!

Morris-Lecar network: results, PRC



▬ true PRC

▬ 10th iteration

Conclusions

- Robust reconstruction of the network structure already for several hundreds of spikes
- Works if the network does not synchronize
- If the coupling is not weak enough: the network reconstruction remains correct, the PRC is amplitude-dependent
- Error of the phase estimation increases with the number of spikes ===> the reconstruction may fail for $\omega_i/\omega \gg 1$
- We need some variability in the drive: the reconstruction may fail for very sparse networks where periodic nodes can be found (however, noise helps here!)

Conclusions II

- Reference: Phys. Rev. E **96**, 012209 (2017)

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COSMOS

Complex Oscillatory Systems:
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Innovative Training Network
European Joint Doctorate

Thank you for your attention!