Como 2017

Explosive synchronization in complex networks

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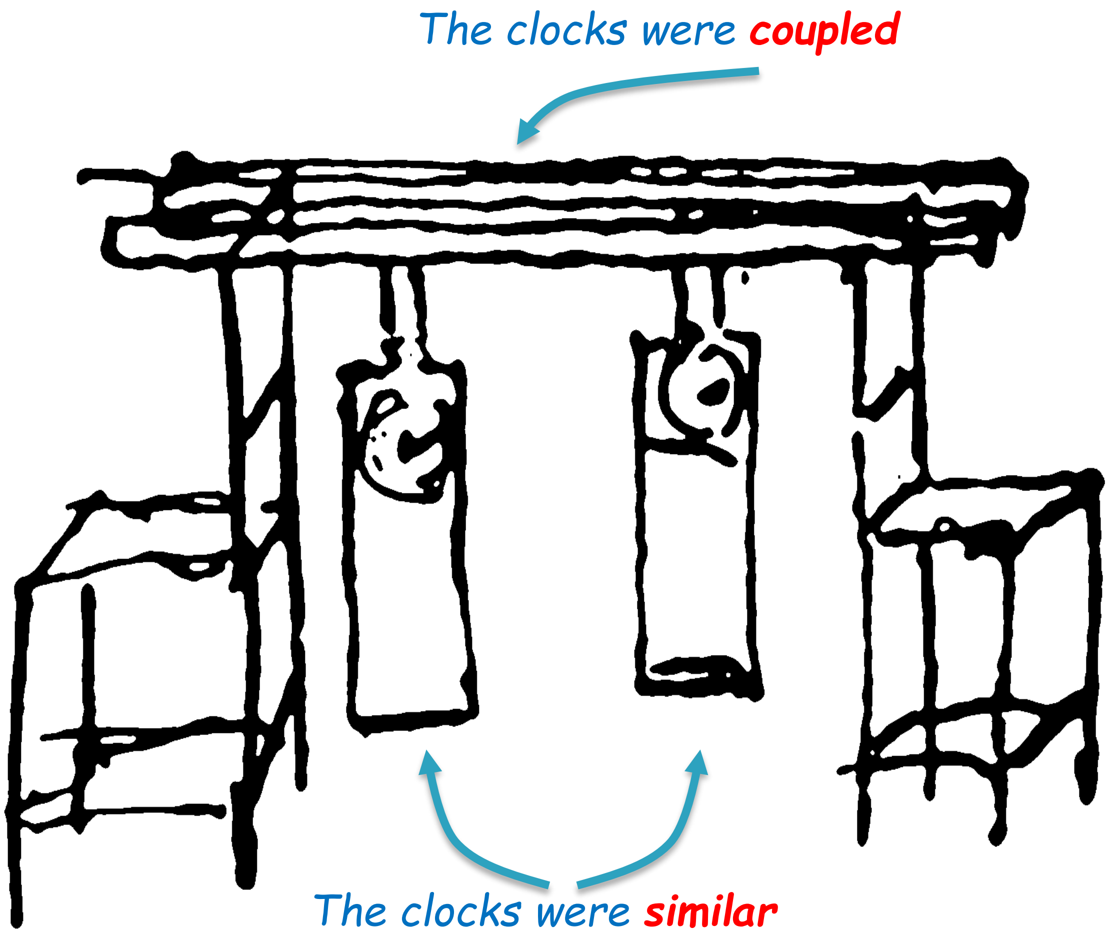
Explosive

Synchronization

in

Complex Networks

The *sympathetic* clocks of Huyghens

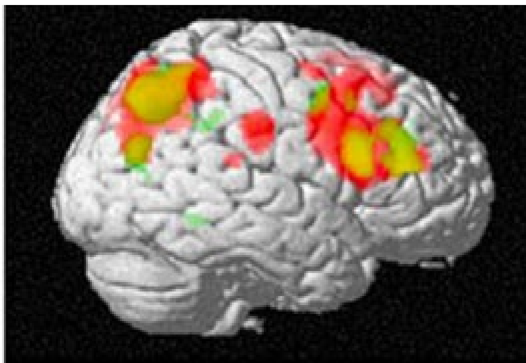


Christiaan Huyghens (1629-1695) discover what he called "an odd kind of sympathy" between the clocks: regardless of their initial state, both *adopted the same rhythm*

Huygens correctly attributed the synchrony ***to tiny forces transmitted by the wooden beam*** from which they were suspended.

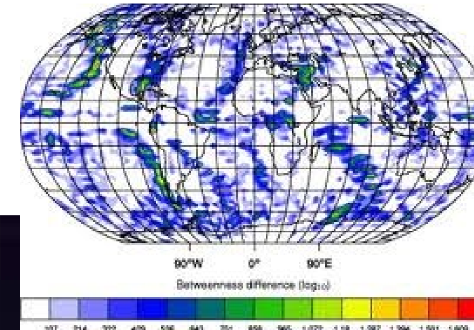
OK, things synchronize. So what?

Synchrony happens to be the main mechanism *for regulating the dynamics and transmit information* **in natural ensembles**…



Brain dynamics

World clima ?



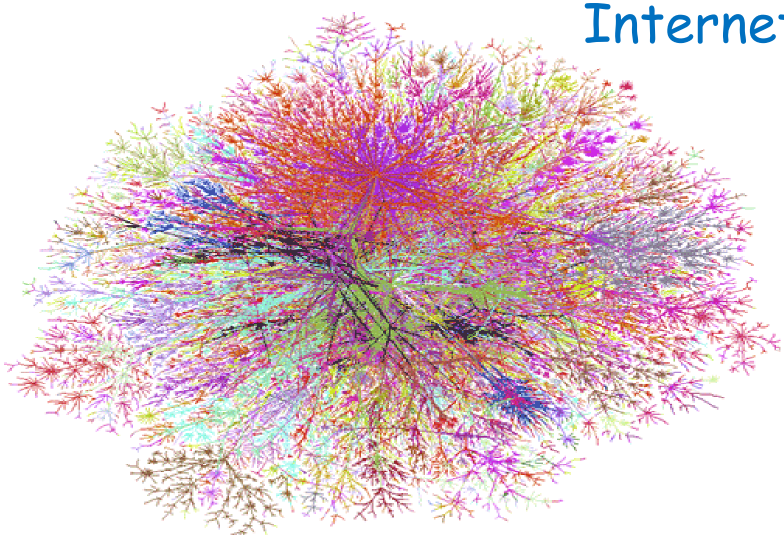
Heart beating

Animal behaviour

Things that synchronize

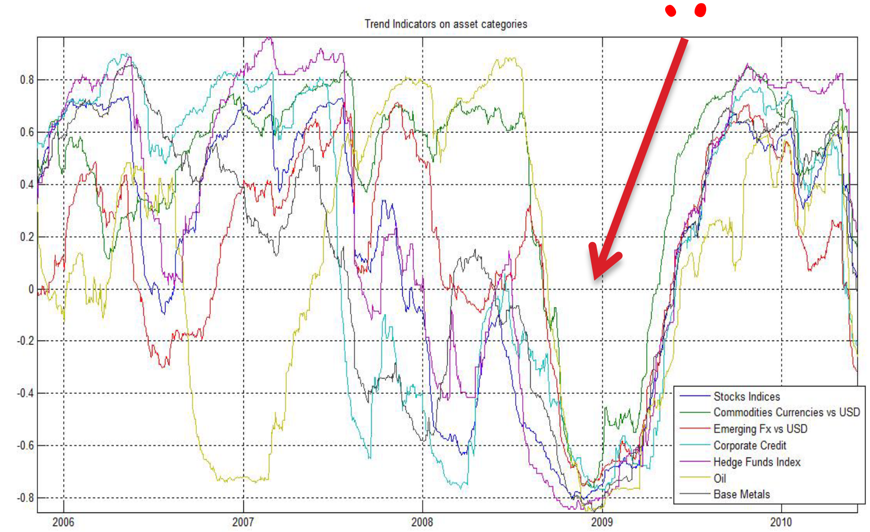
…and in social or artificial ones

Internet



Financial markets

!?



Human behaviour

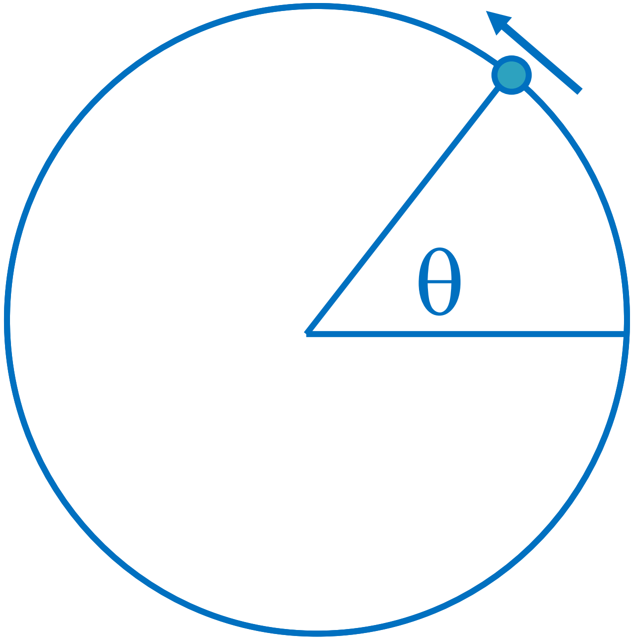
Power grids

Keeping the essencial: the Kuramoto oscillator

- *A periodic oscillator with an intrinsic (or* natural*)* **frequency** ω_n.

- The evolution of each oscillator n is described only by its **phase** θ_n such that

$$\dot{\theta}_n = \omega_n$$



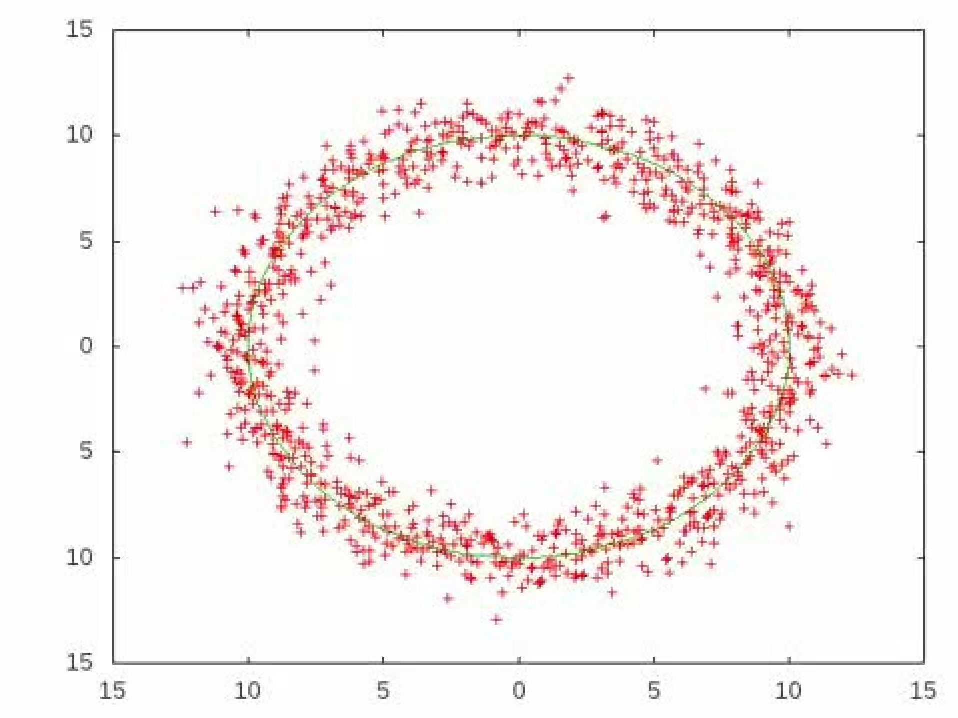
- We are interested in **heterogeneous ensembles**, so we assume the ω_n frequencies are different, randomly picked from an (usually known) distribution $g(\omega)$

- *A large ensemble of N oscillators*

Kuramoto ensemble: all-to-all coupling

Sinusoidal ***all-to-all*** coupling.

$$\dot{\theta}_n = \omega_n$$

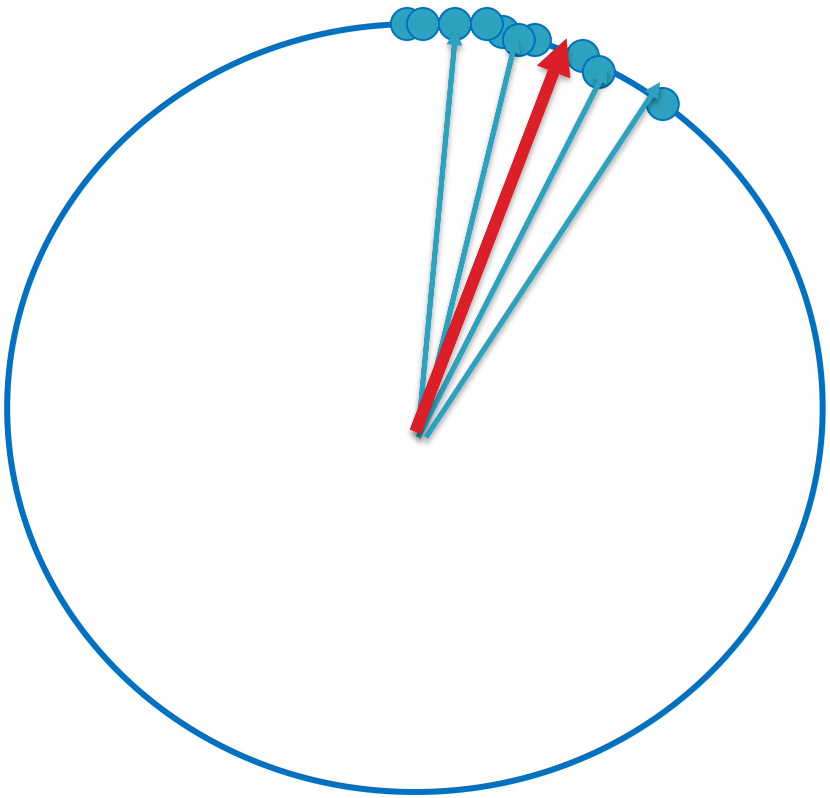


If d is high enougth, phases lock and oscillators frequency converge to the average < ω_n>

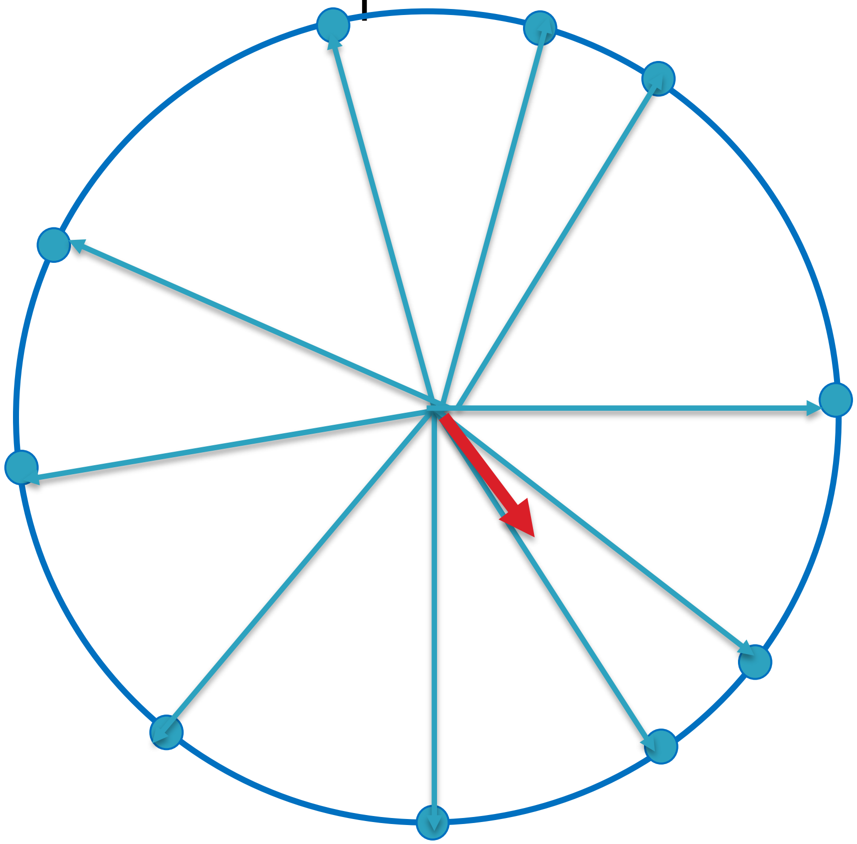
(N=1000)

Measuring synchronization: Kuramoto order parameter

$$r = \frac{1}{N}\left|\sum_{m=1}^{N} \exp(i\theta_m)\right|$$

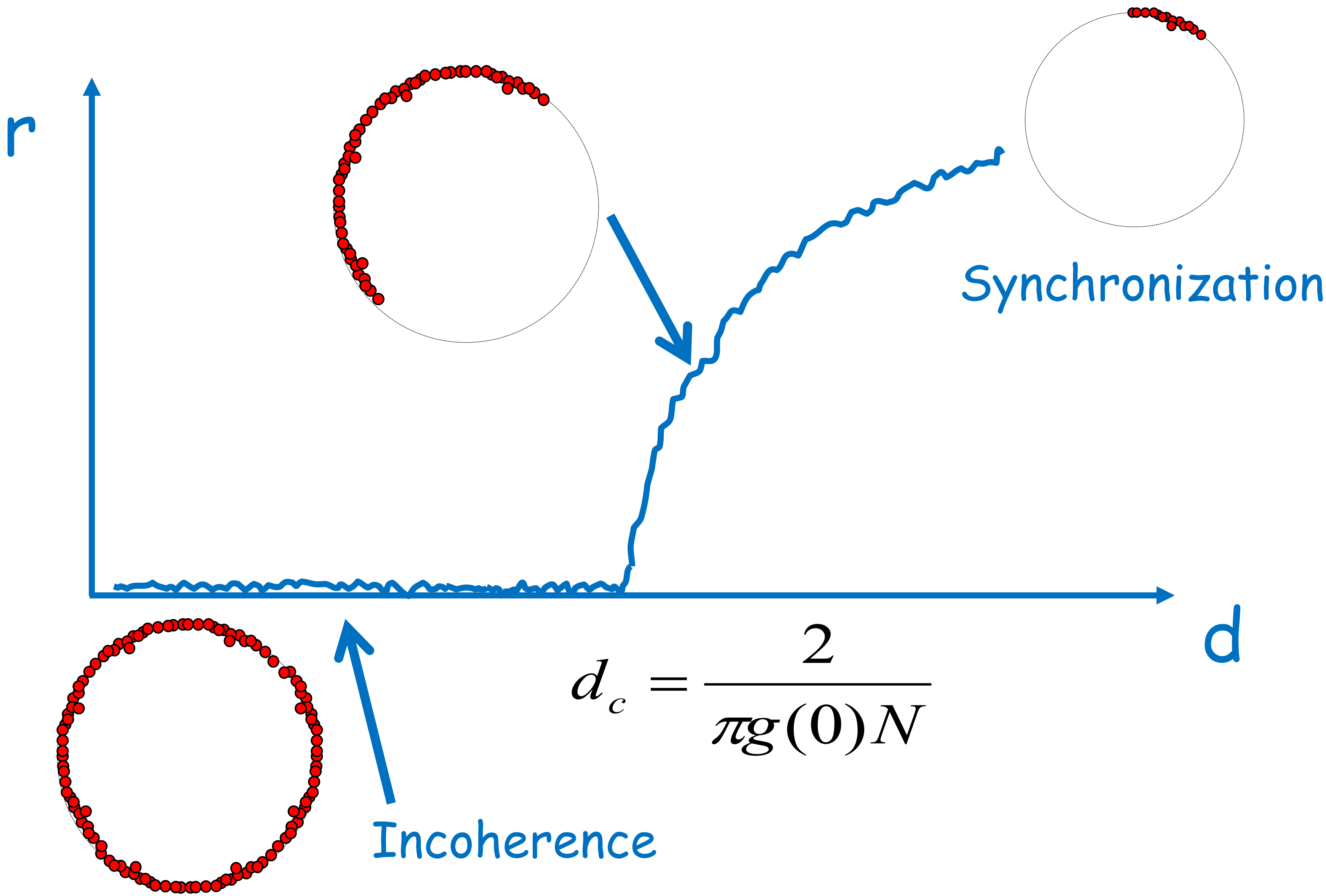


$r \approx 1$



$r \approx 0$

Path to synchrony in the Kuramoto model



$$d_c = \frac{2}{\pi g(0) N}$$

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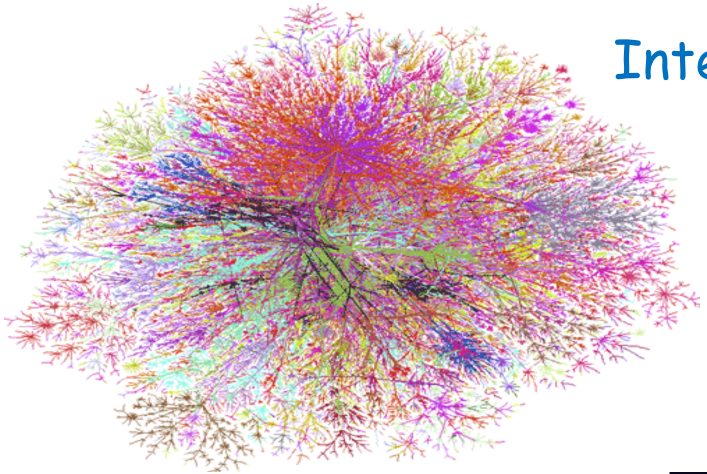
Synchronization

in

Complex Networks

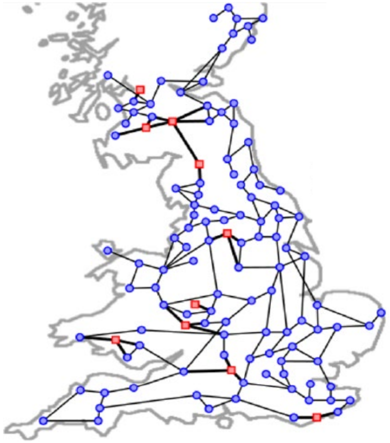
Why complex netorks?

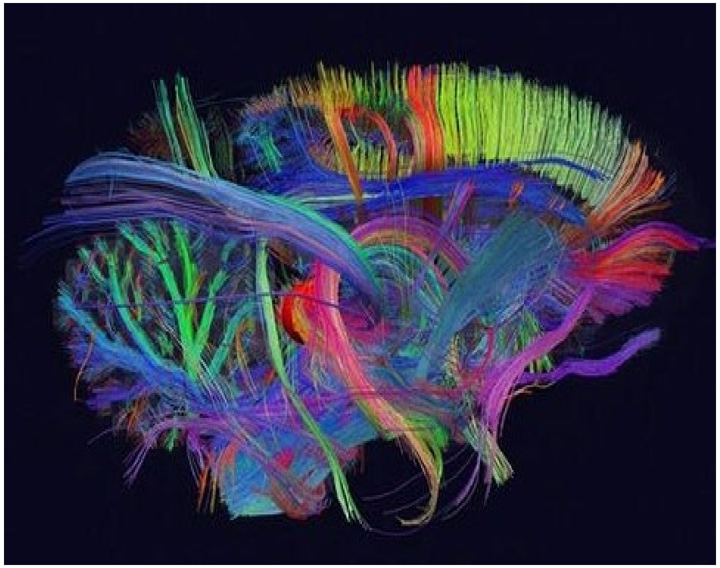
Most of the systems where synchronization is important are complex networks



Internet

Power grid





Brain

Next step: shyncronization of Kuramoto units in complex networks

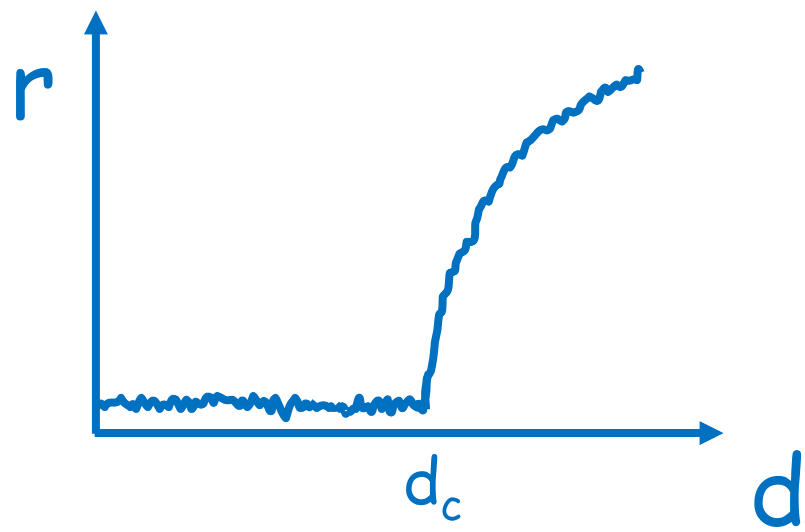
In order to study the effect of a network on the emergence of synchronization, we will maintain the simple phase dynamics, but will introduce a complex network in the problem

$$\dot{\theta}_n = \omega_n + d\sum_{m=1}^{N} A_{nm} \sin(\theta_m - \theta_n)$$

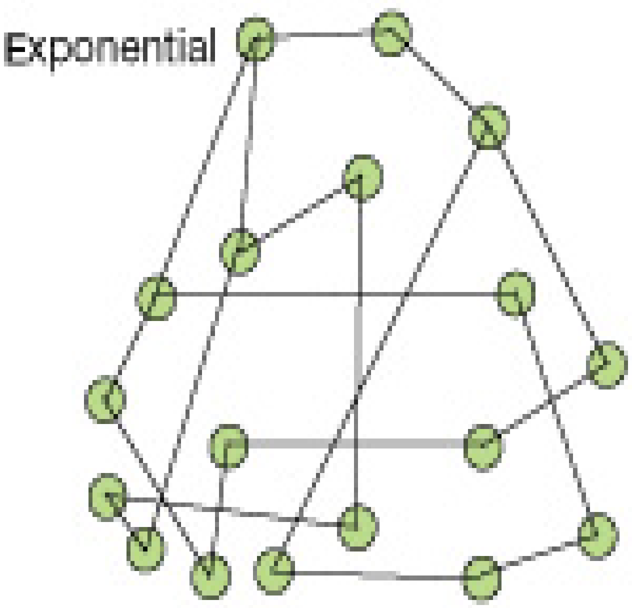
If m links to *n* ⟺ $A_{nm} = 1$

(else $A_{nm} = 0$)

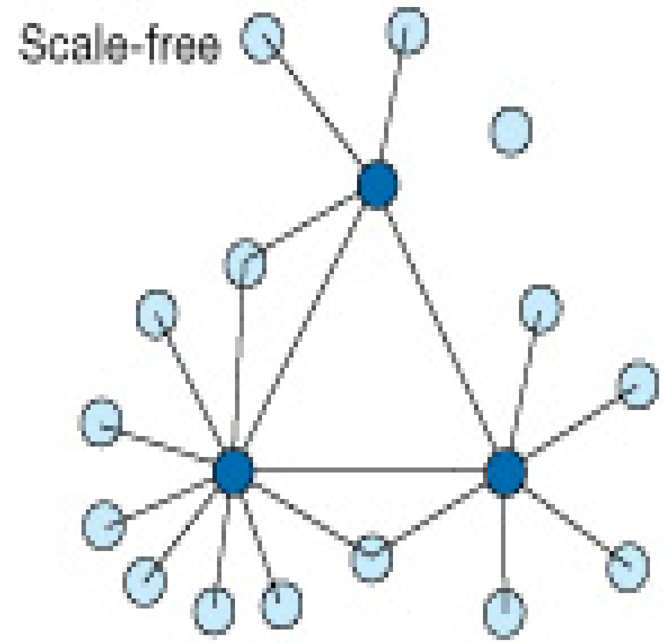
$$d_c = \frac{2}{\pi g(0)} ?$$



Stucture matters



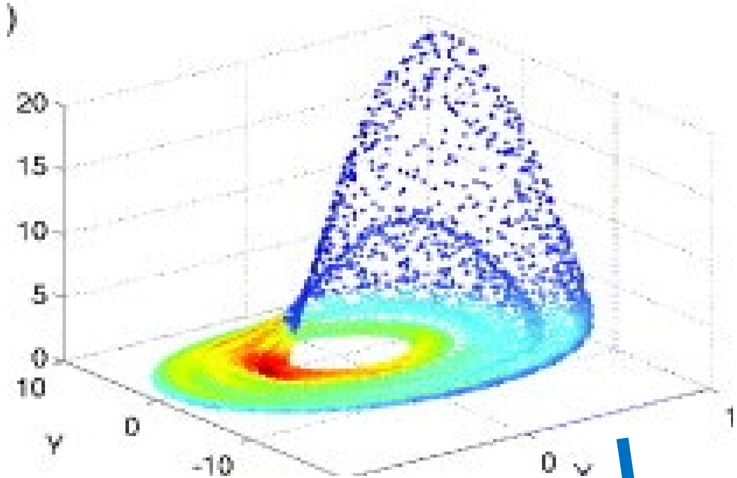
$$d_c > d_c$$

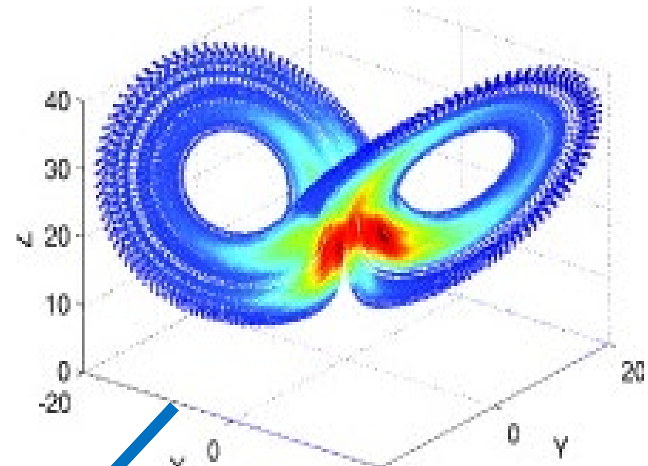


For a given number of nodes and connections,
heterogeneous networks synchronizes easier

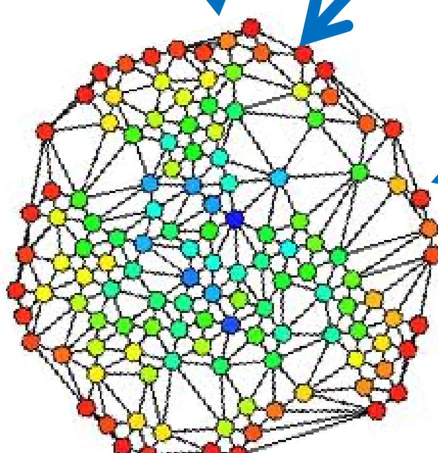
Stucture matters !

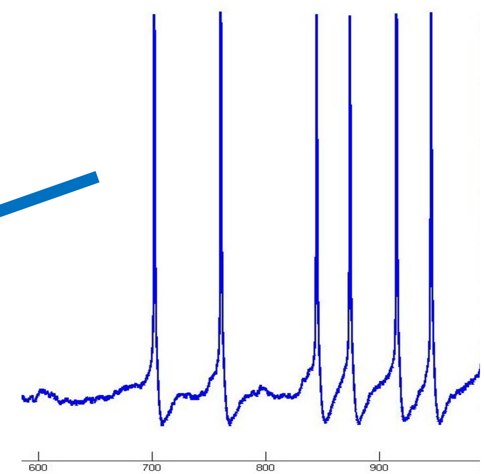
Complex dynamics in complex networks





Synchronization
strongly
depends on
both topology
AND dynamics





…in a way too long
for telling you today !

Explosive

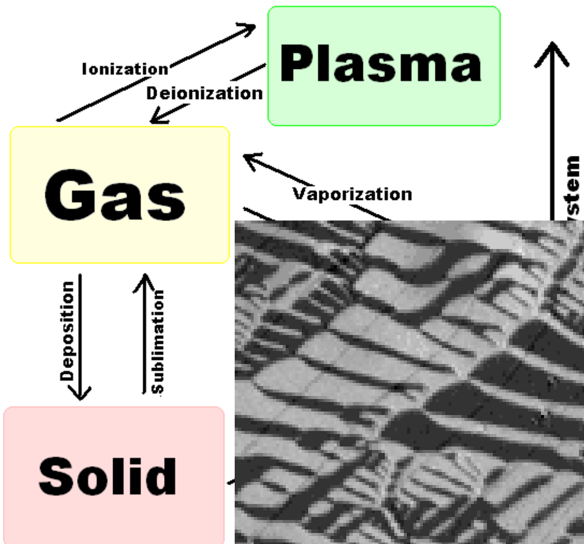
Synchronization

in

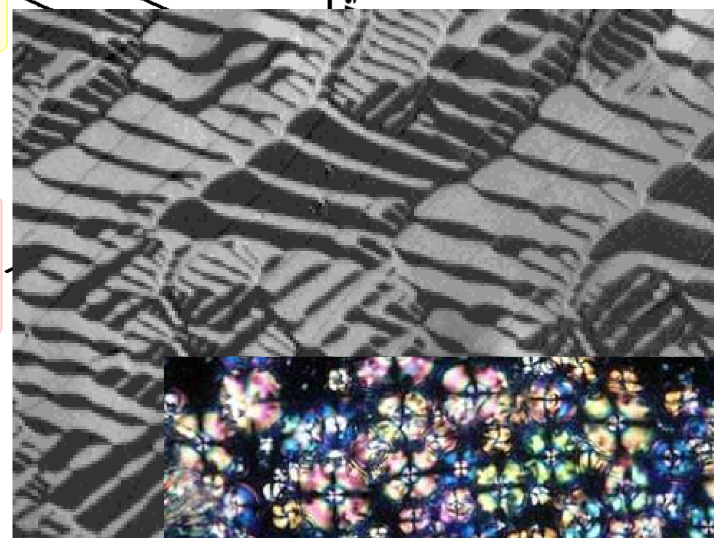
Complex Networks

Phase transitions

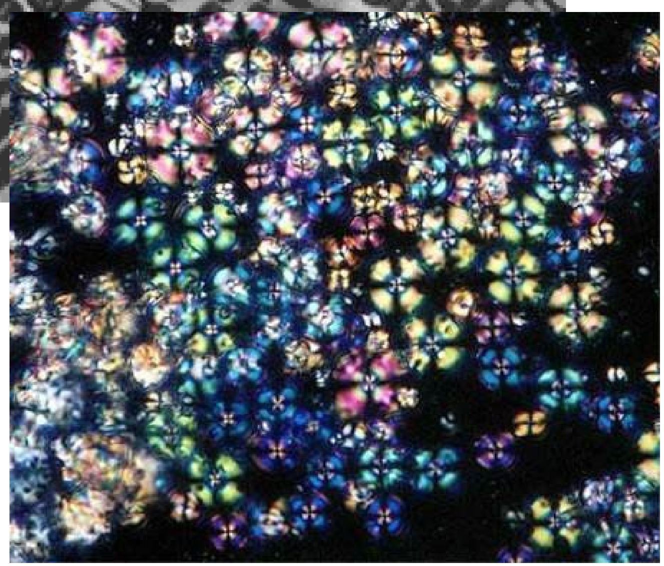
Changes between different states of organization in a system



States of matter



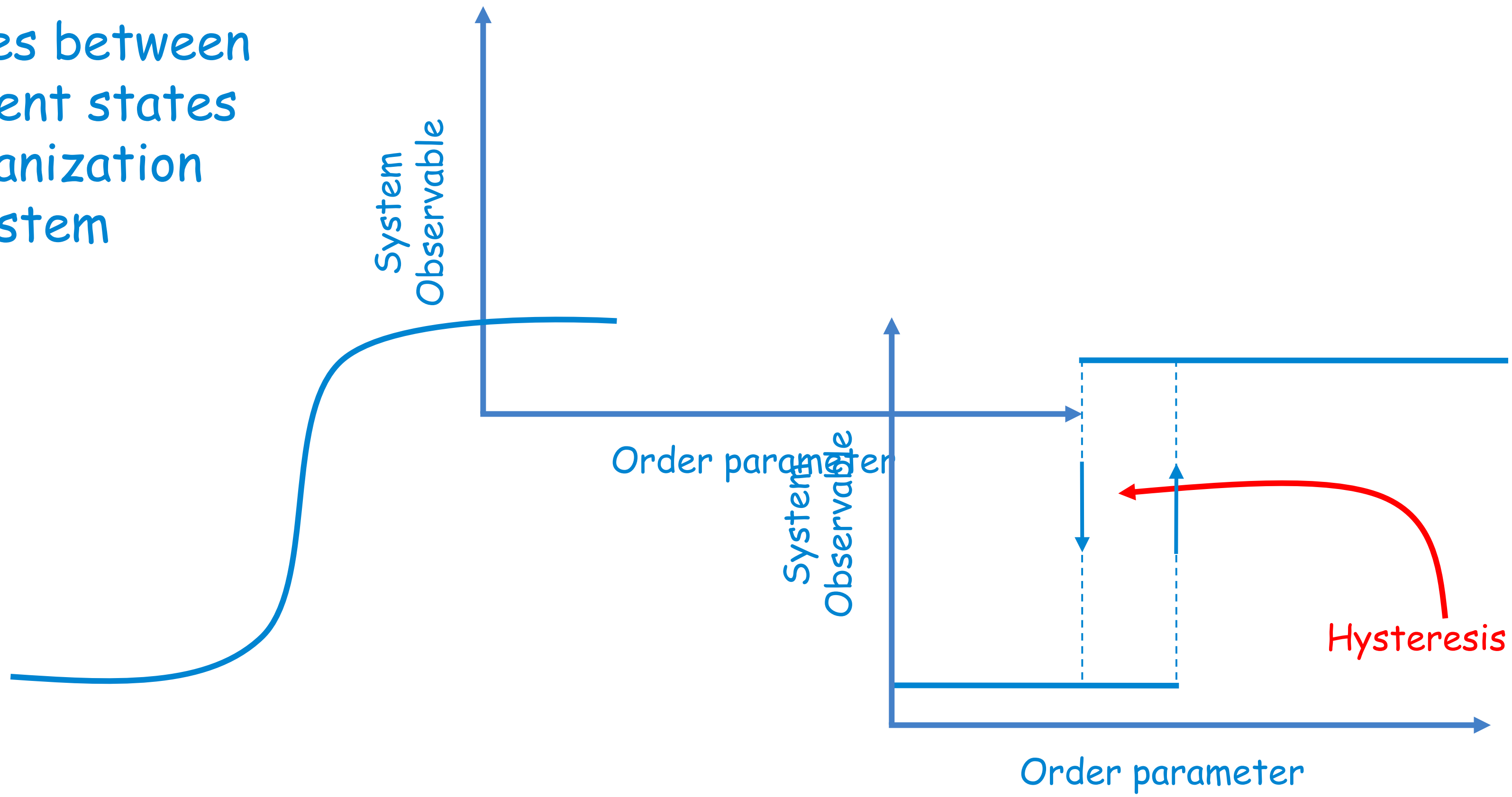
Magnetization



Liquid cristal nematic state

Remembering phase transitions

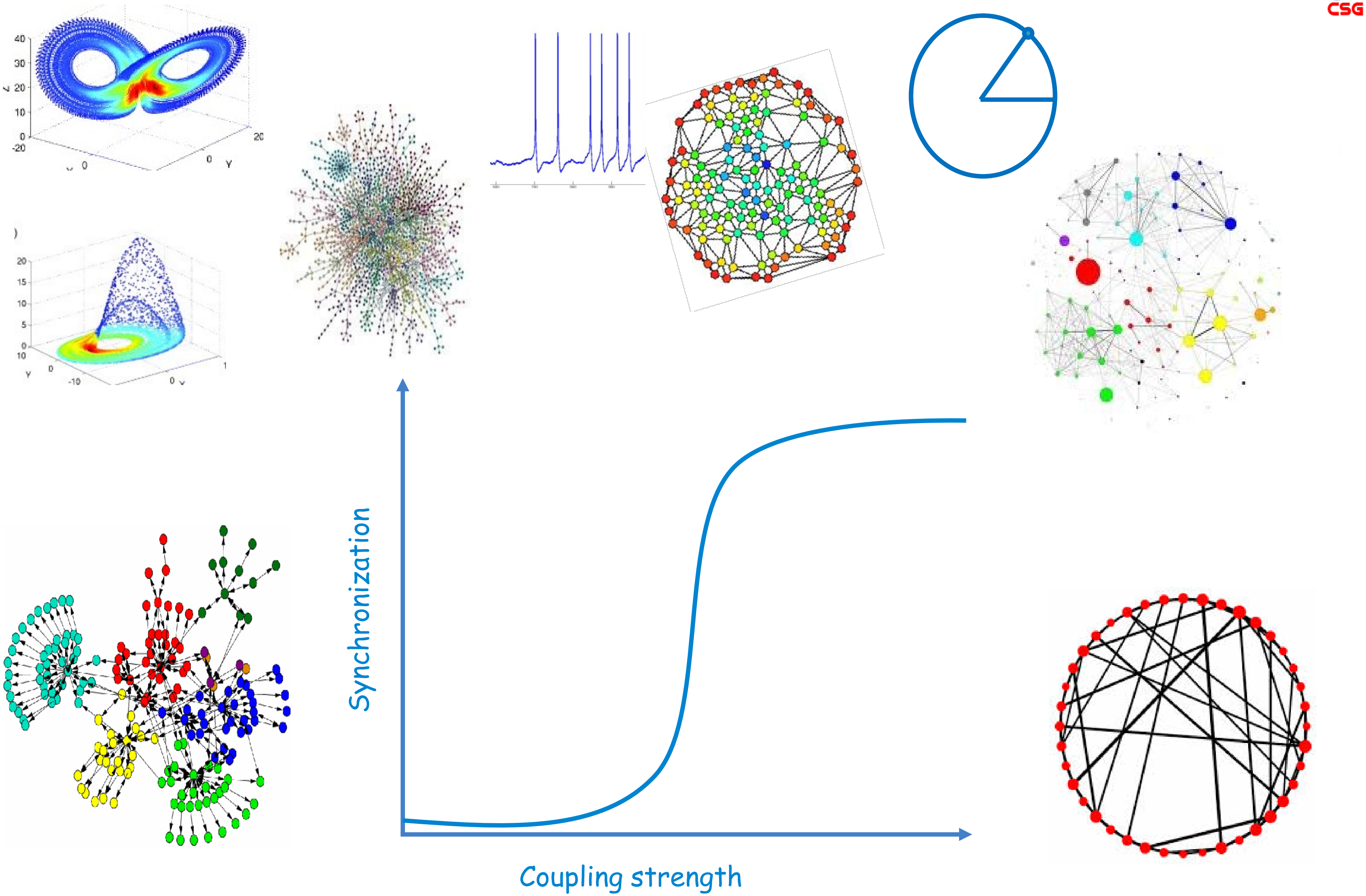
Changes between different states of organization in a system



Second order PT

First order PT

Transitions to synchrony in complex networks



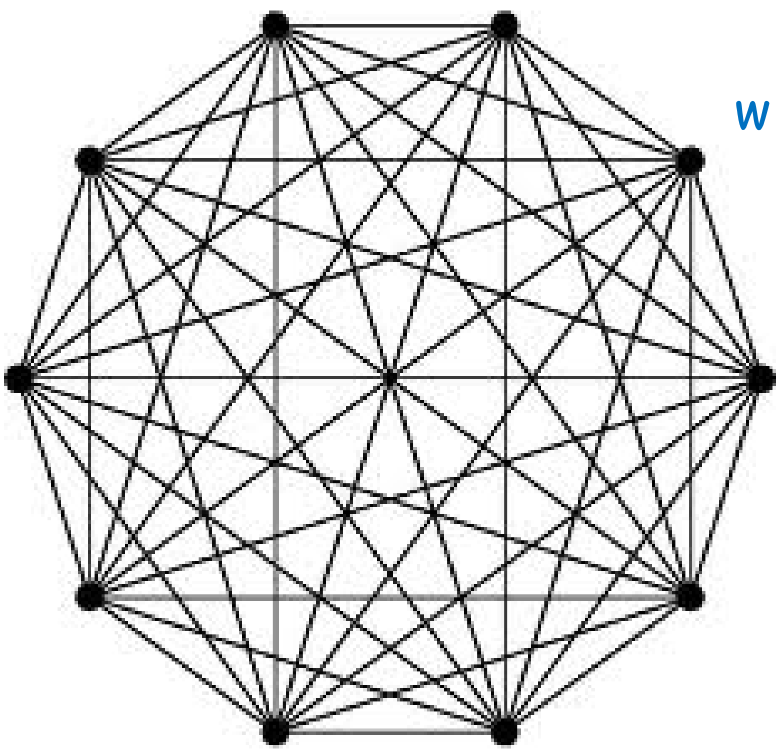
Case 1: Kuramoto model with a special frequency distibution

PHYSICAL REVIEW E 72, 046211 (2005)

Thermodynamic limit of the first-order phase transition in the Kuramoto model

Diego Pazó*
Max-Planck-Institut für Physik komplexer Systeme, Nöthnitzer Straße 38, 01187 Dresden, Germany

Full Kuramoto model **with *equispaced* frequencies**



$$\omega_j = -\gamma + \frac{\gamma}{N}(2j-1)$$

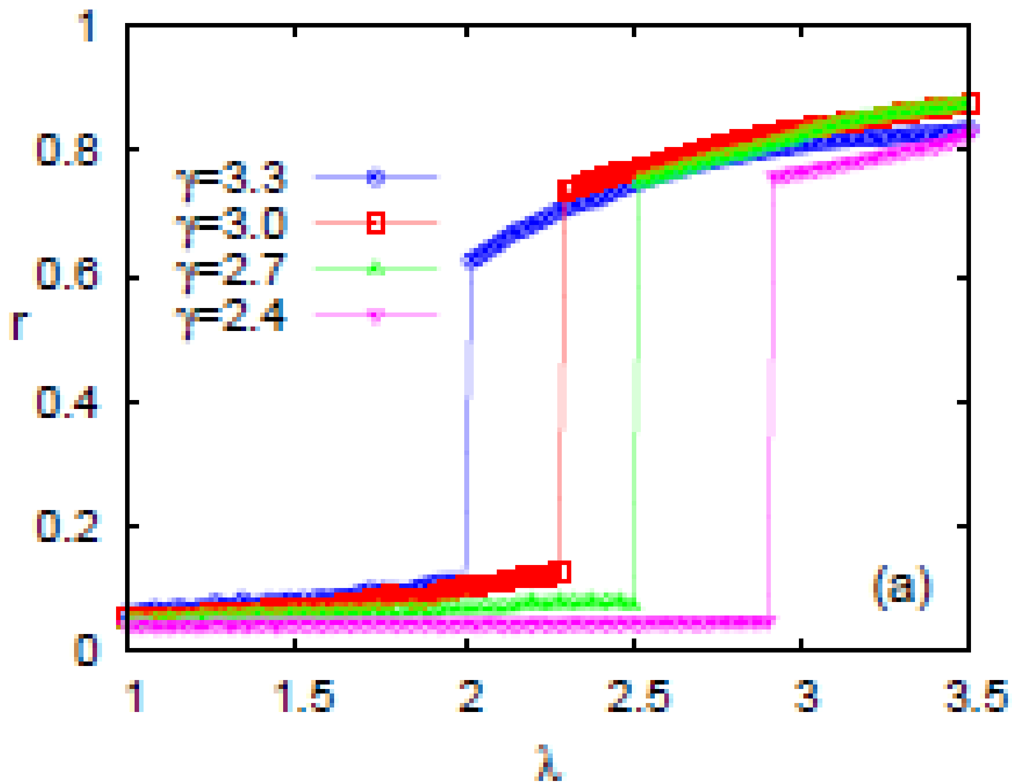
Case 2: SF + degree-frequency correlation

PRL 106, 128701 (2011) PHYSICAL REVIEW LETTERS week ending 25 MARCH 2011

Explosive Synchronization Transitions in Scale-Free Networks

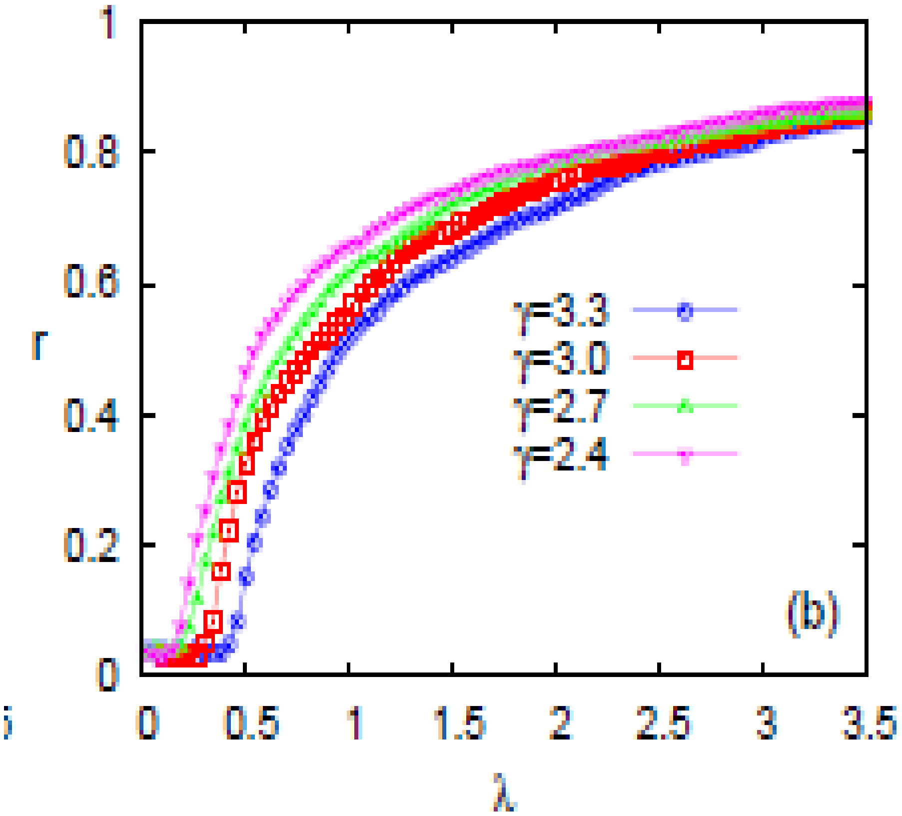
Jesús Gómez-Gardeñes,[1,2,*] Sergio Gómez,[3] Alex Arenas,[2,3] and Yamir Moreno[2,4]

SF networks of **Kuramoto** oscillators where ω_i proportional to k_i

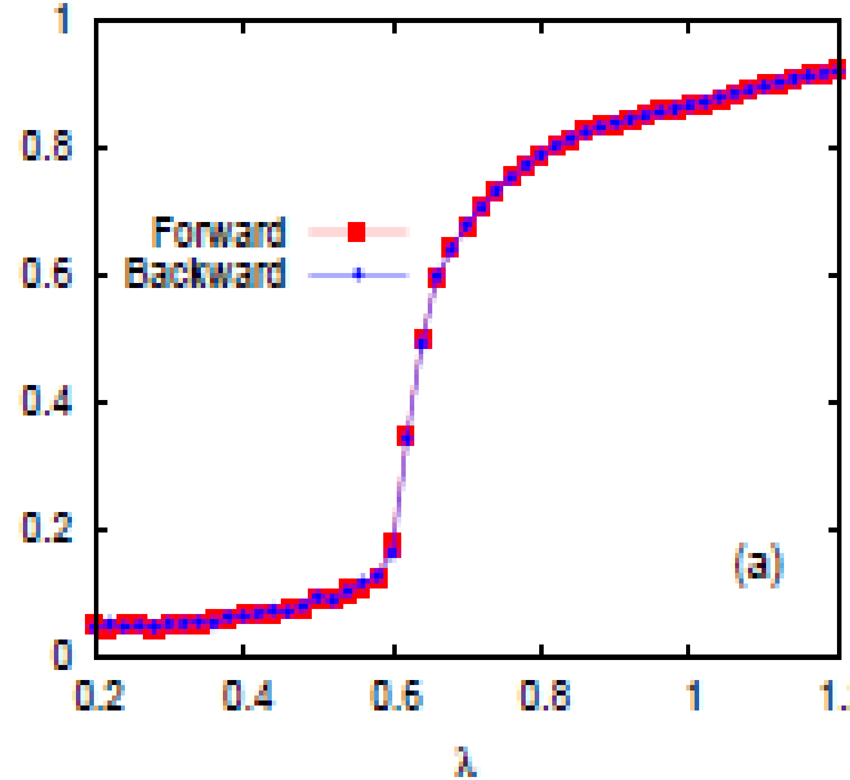


Case 2: SF network + degree-frequency correlation

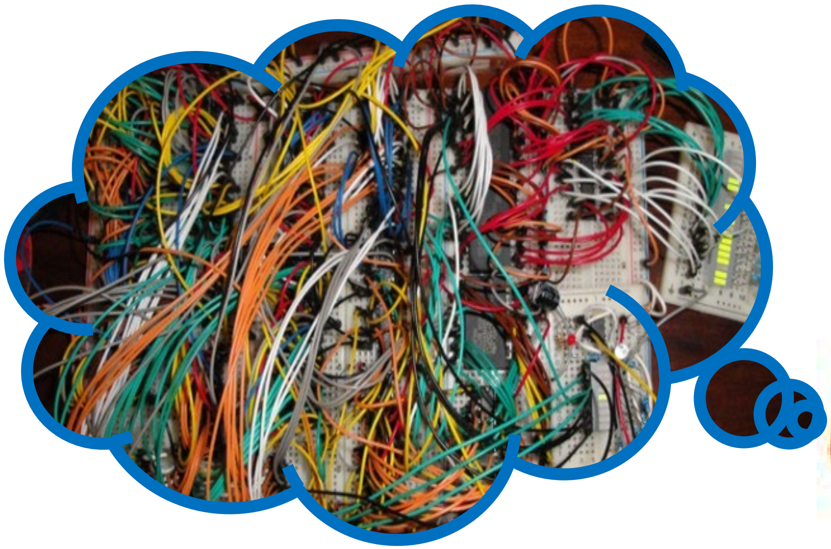
SF but uncorrelated ω_i , k_i



Correlated ω_i , k_i but random network



(What the coffe-breaks are useful for…)



Wouldn't be *cool* if this could be done

EXPERIMENTALLY ?

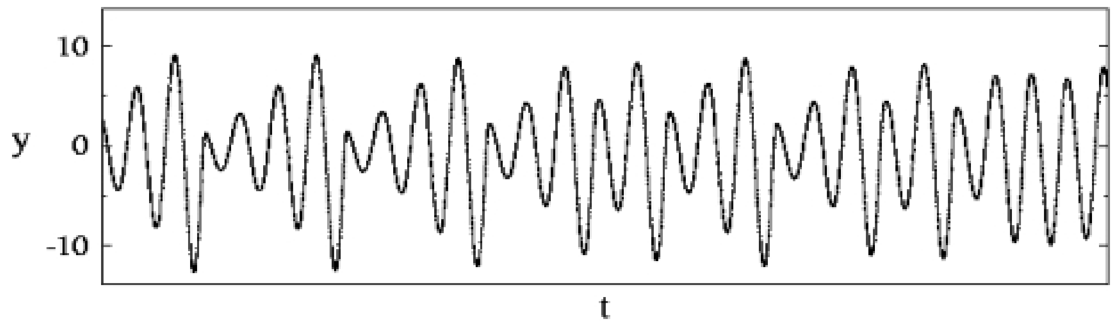
The model: Rössler oscillator

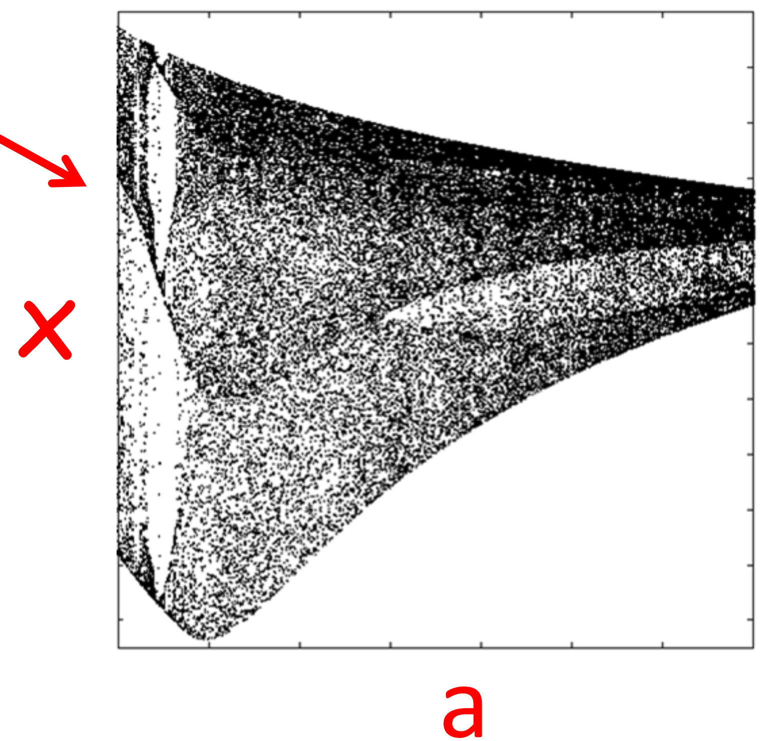
Even more fun: Can it be done experimentally

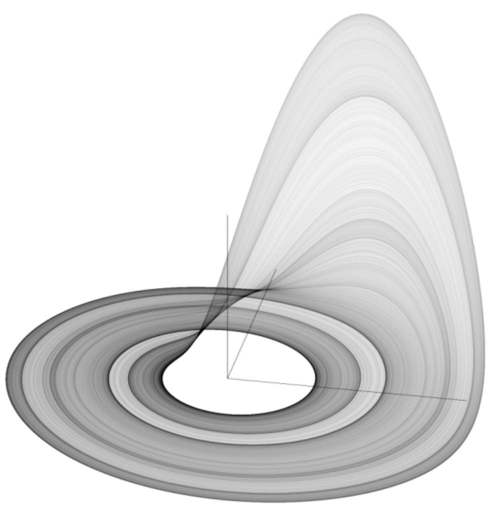
$$\dot{x} = -\omega y - z$$

$$\dot{y} = \omega x + ay$$

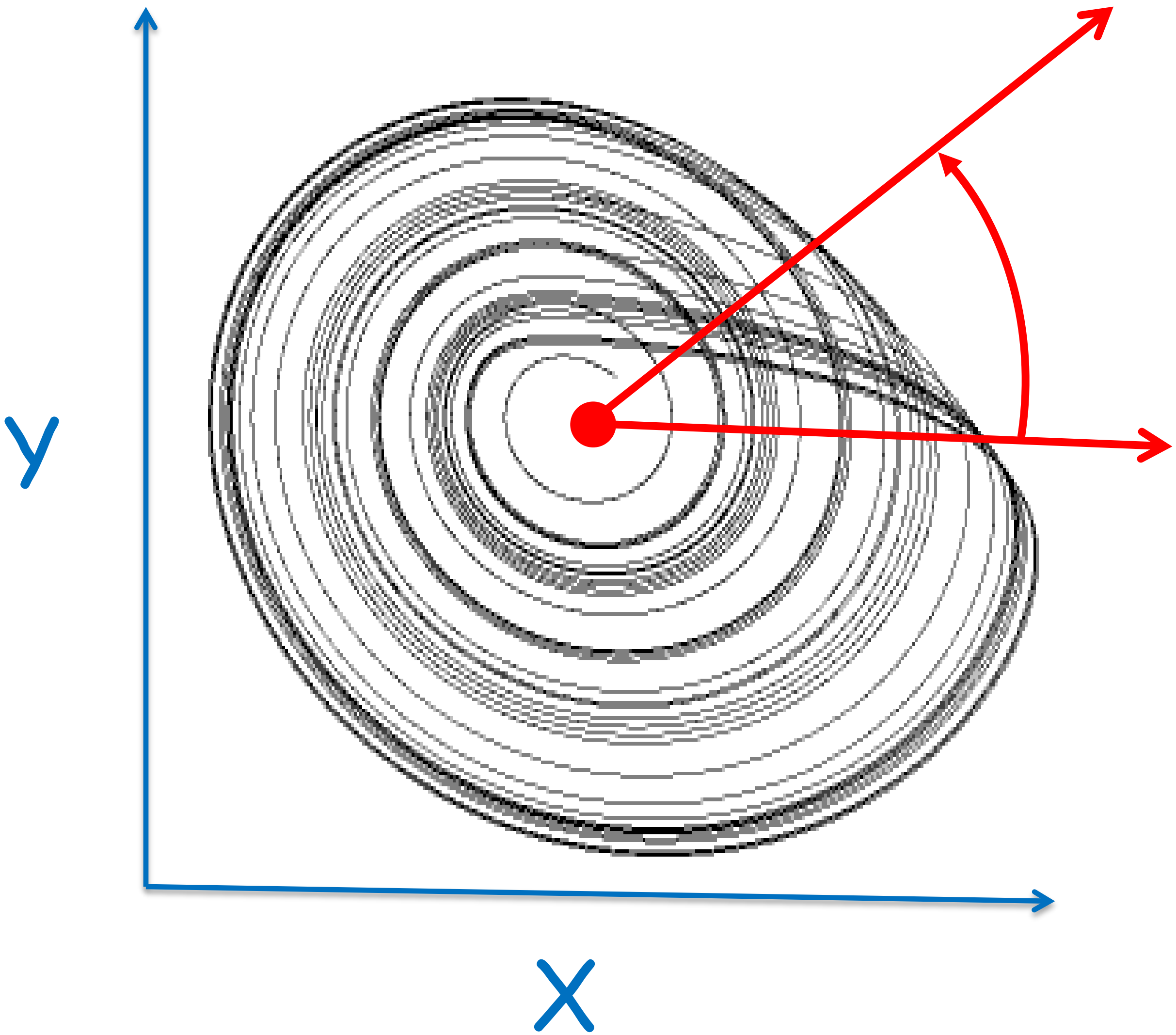
$$\dot{z} = b + z(x - c)$$







Phase and amplitude in the chaotic Rössler oscillator



Phase

$$\theta = artag\left(\frac{y}{x}\right)$$

The *actual* model: piecewise Rössler oscillator

$$\dot{x}_i = -\alpha_i\left(\Gamma x_i + \beta y_i + \lambda z_i\right) + d\sum_{j=1}^{N} a_{ij}(x_j - x_i)$$

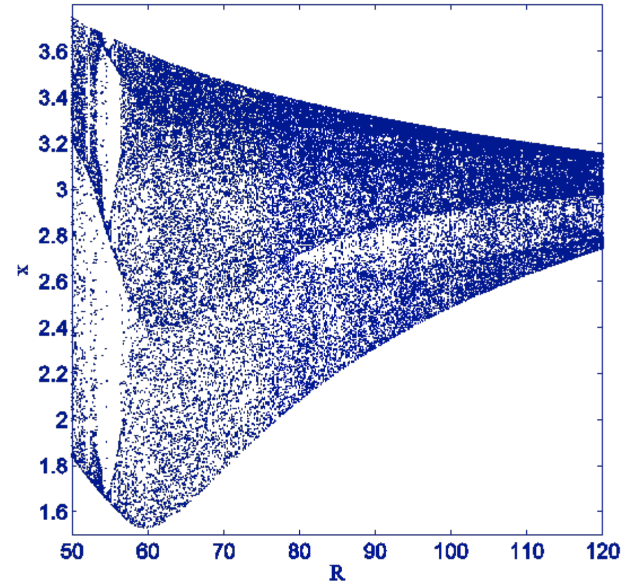
$$\dot{y}_i = -\alpha_i\left(-x_i + \left(m - \frac{n}{R}\right)y_i\right)$$

$$\dot{z}_i = -\alpha_i\left(-g(x_i) + \lambda z_i\right)$$

piecewise part

Frequency control in a very large range

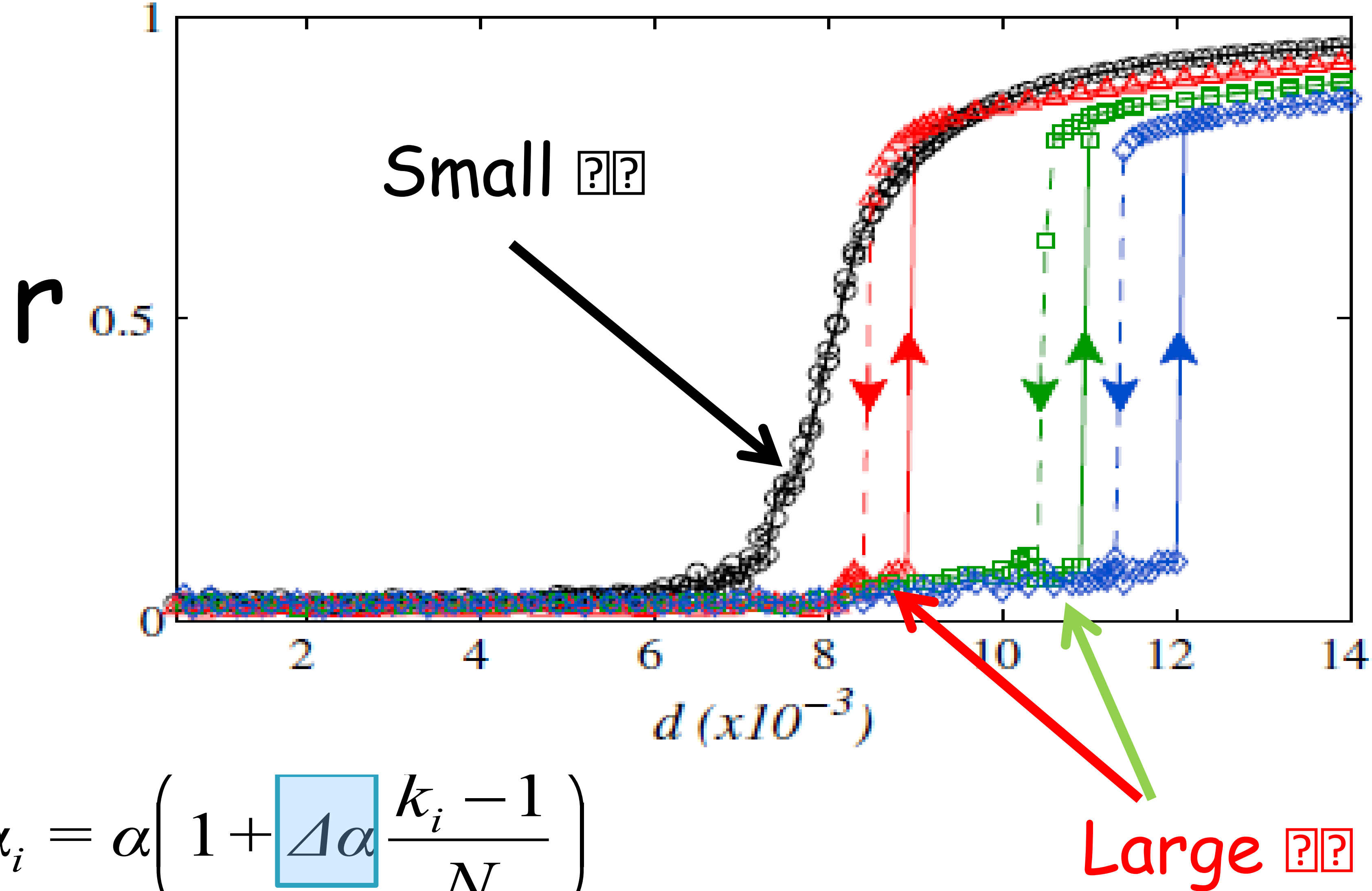
$$\alpha_i = \alpha\left(1 + \Delta\alpha\,\frac{k_i - 1}{N}\right)$$



Dynamical state control

Simulation: explosive phase synchronization

SF network, N=1000, several values of ??

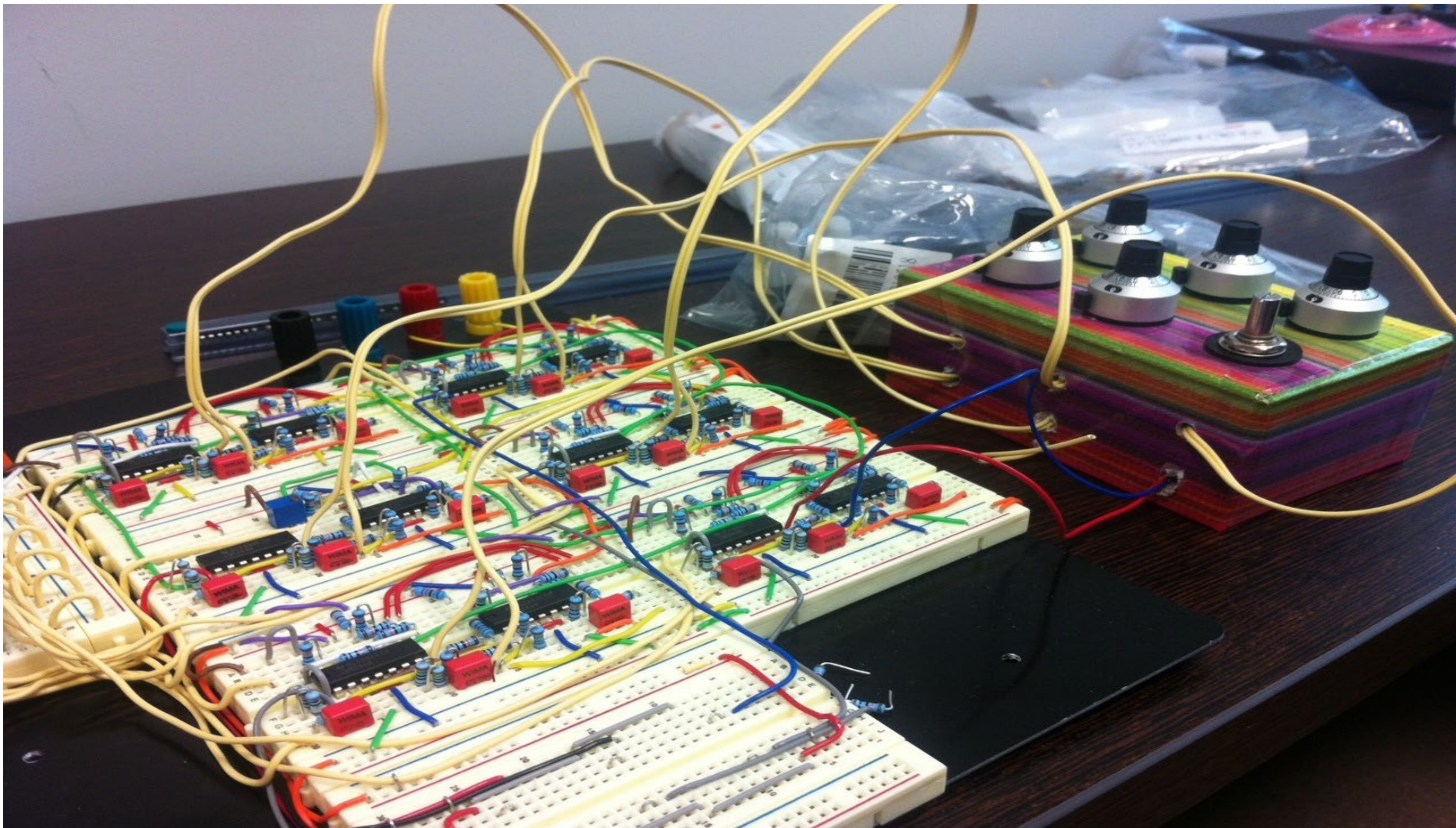


$$\alpha_i = \alpha\left(1 + \Delta\alpha \frac{k_i - 1}{N}\right)$$

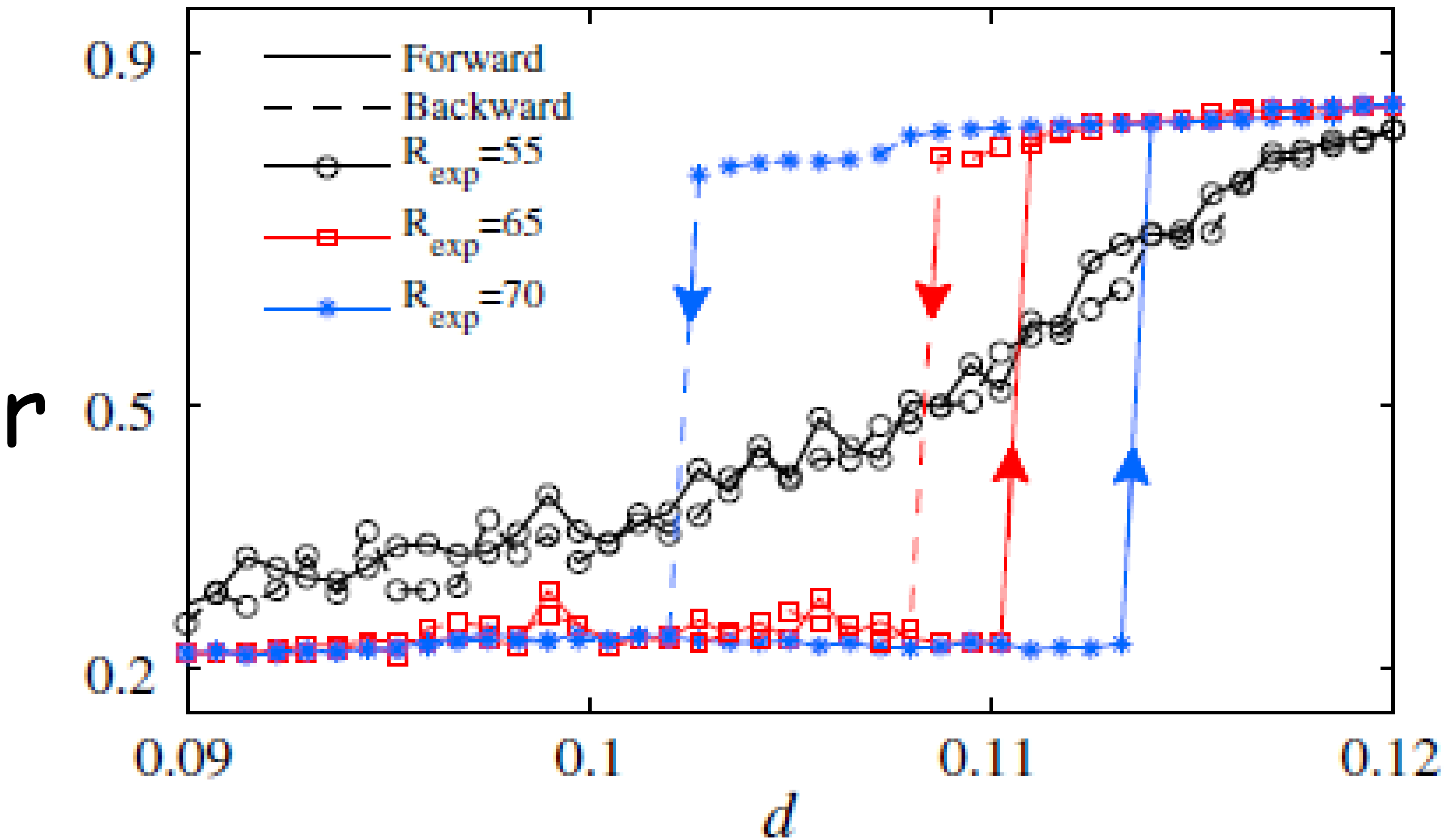
The experiment: star network

N= 5 + 1 with common R parameter ➡ same dynamical state

- Fast node N1 ⍰$_1$= 3333 Hz
- Slow nodes N2…..N6 ⍰$_i$= 2240 ⍰200 Hz



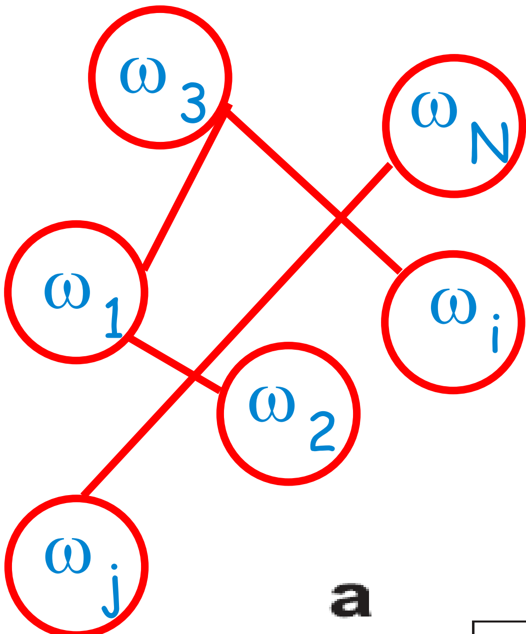
Experimental explosive synchronization



First experimental evidence of first order synchronyzation

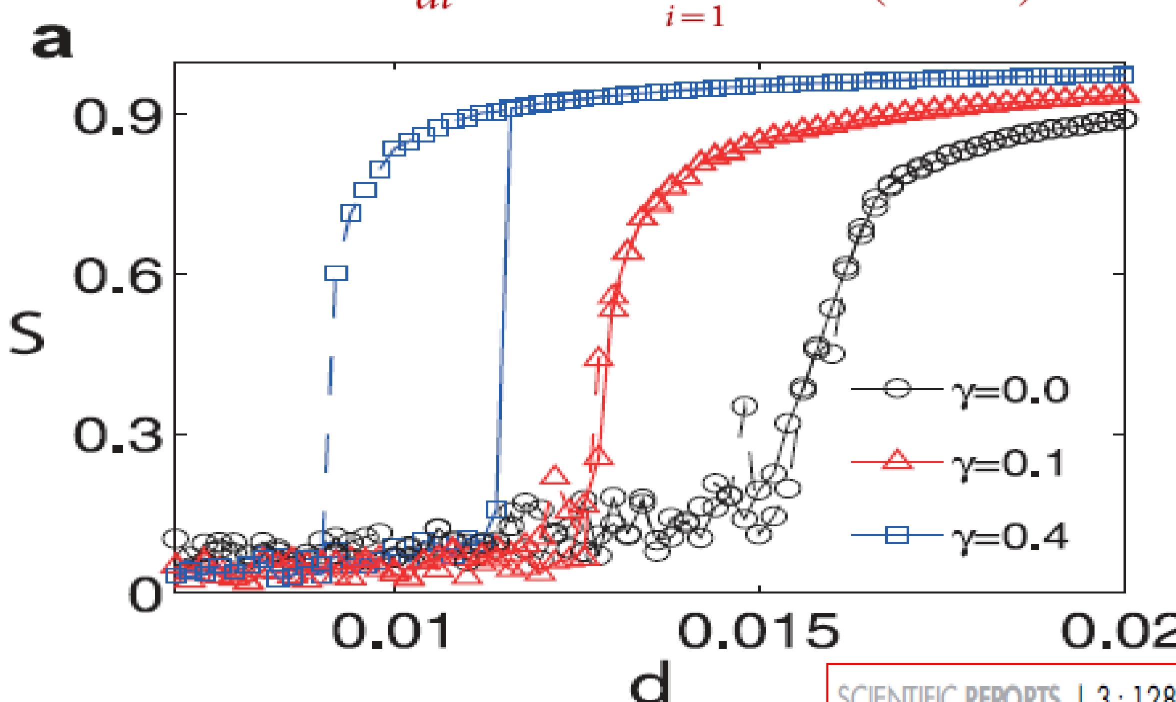
PRL **108**, 168702 (2012)

Making you network to explode I : Gap method



- Distribute frequencies (valid for any g(w))
- Pick a random pair i,j
- Only if $|\omega_i - \omega_j| > \gamma \Rightarrow a_{ij}=1$
- Continue up to construct target network (<k> ?)

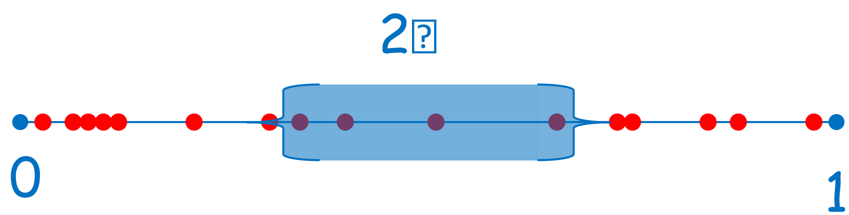
$$\frac{d\phi_i}{dt} = \omega_i + d \sum_{i=1}^{N} a_{ij} \sin\left(\phi_j - \phi_i\right),$$

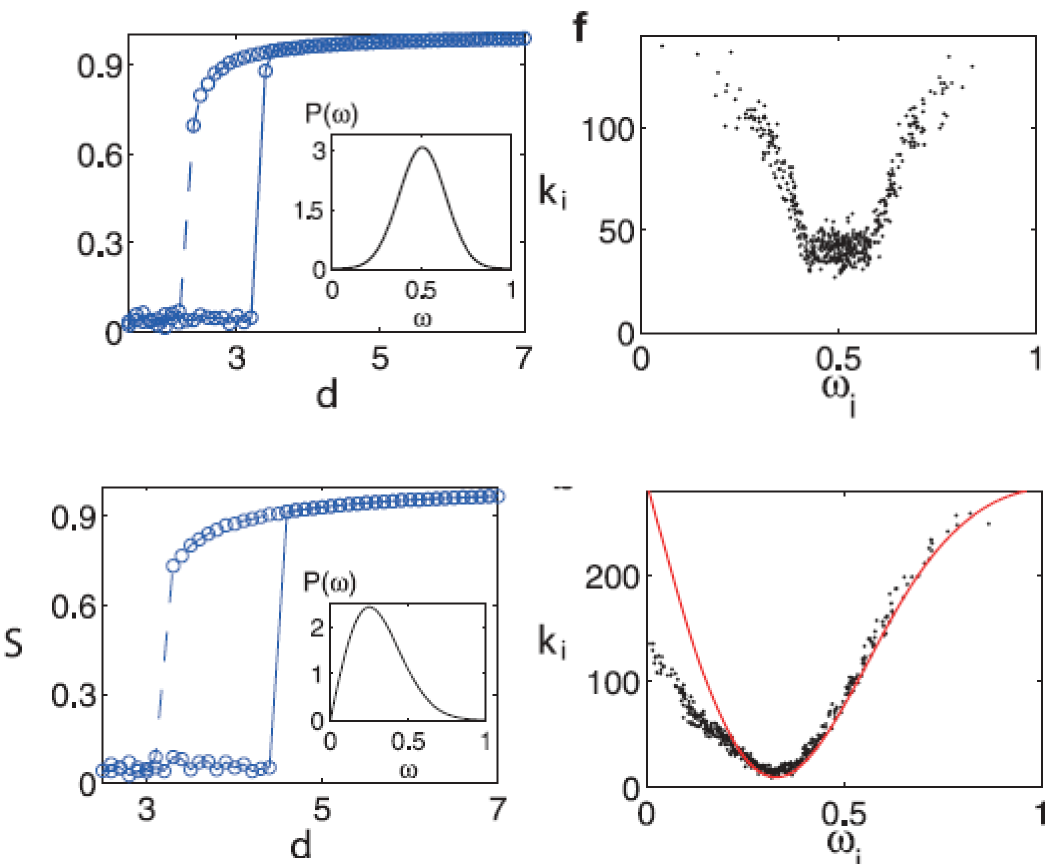


SCIENTIFIC REPORTS | 3 : 1281 | DOI: 10.1038/srep01281

Gap method: emergent k-□ correlation

Probability of node i of being linked depends on $\omega_i \longrightarrow g(\omega)$





… but work equally for regular random networks (all nodes with same k)

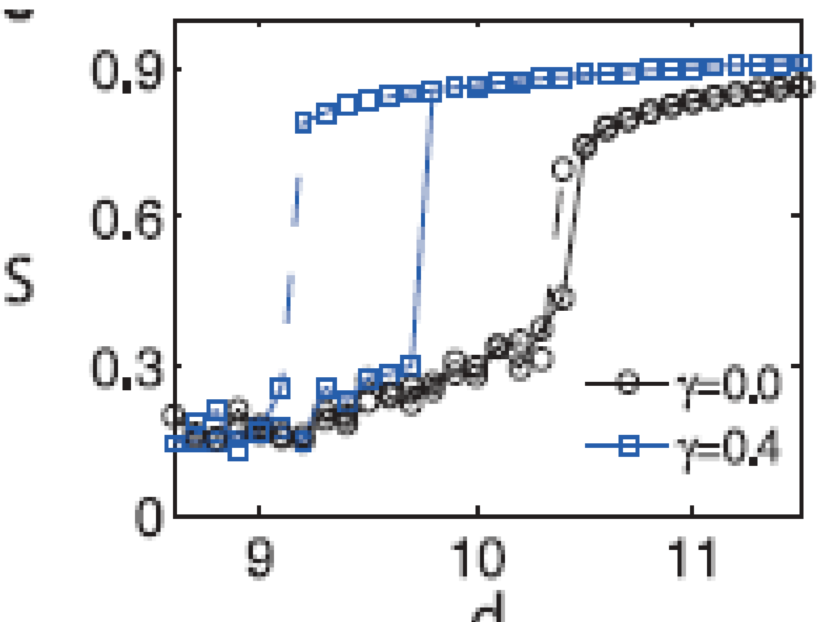
k-□ correlation can emerge, ***but is not a condition***

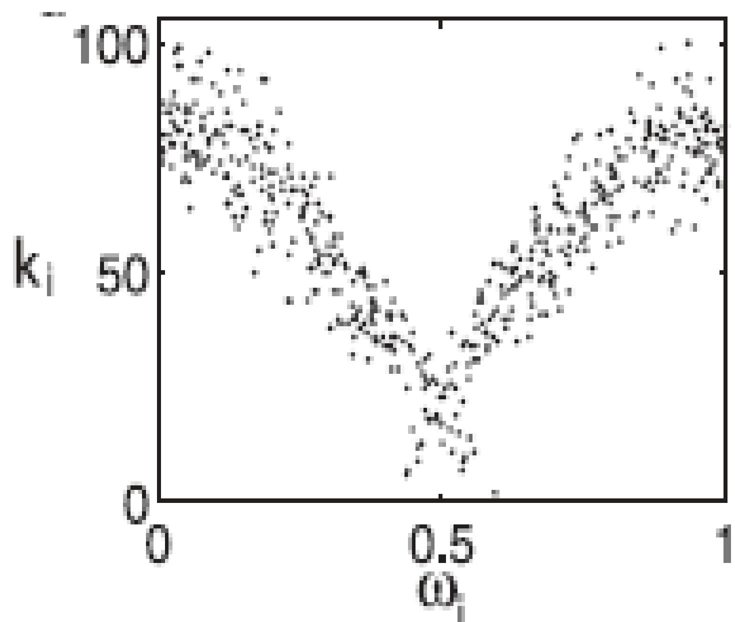
Also works for weaker rules as neighbourhood averaged gap:

$$|\omega_i - \langle \omega_j \rangle| > \gamma$$

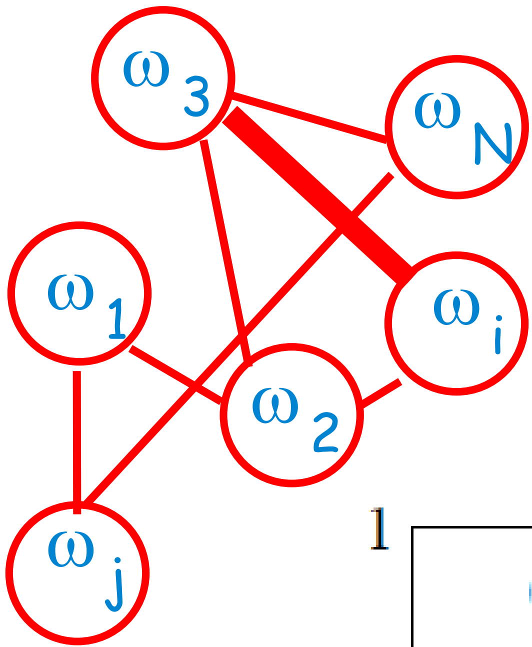
$$|\omega_j - \langle \omega_i \rangle| > \gamma$$

$\Rightarrow a_{ij}=1$



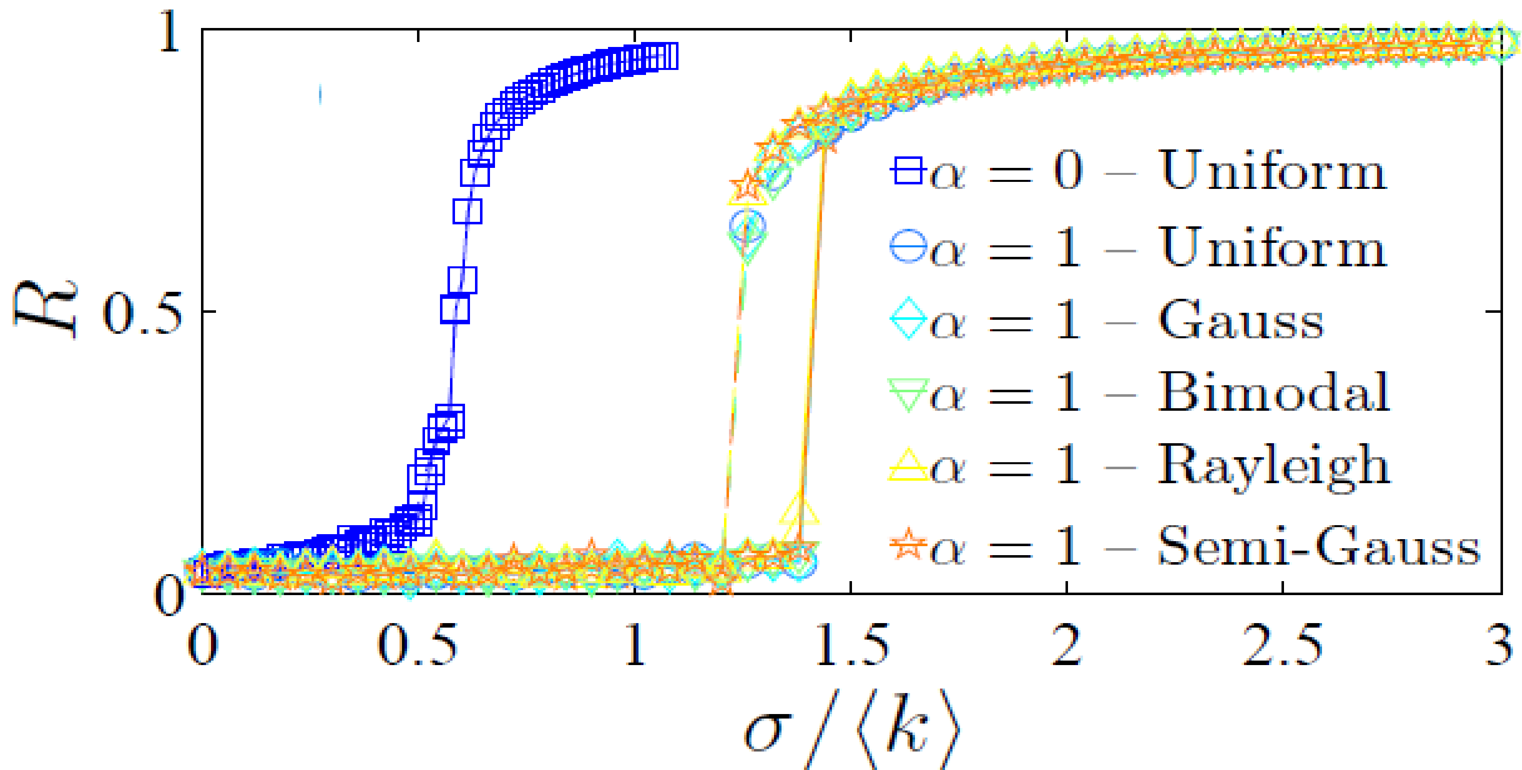


Making your network to explode II : weighting method



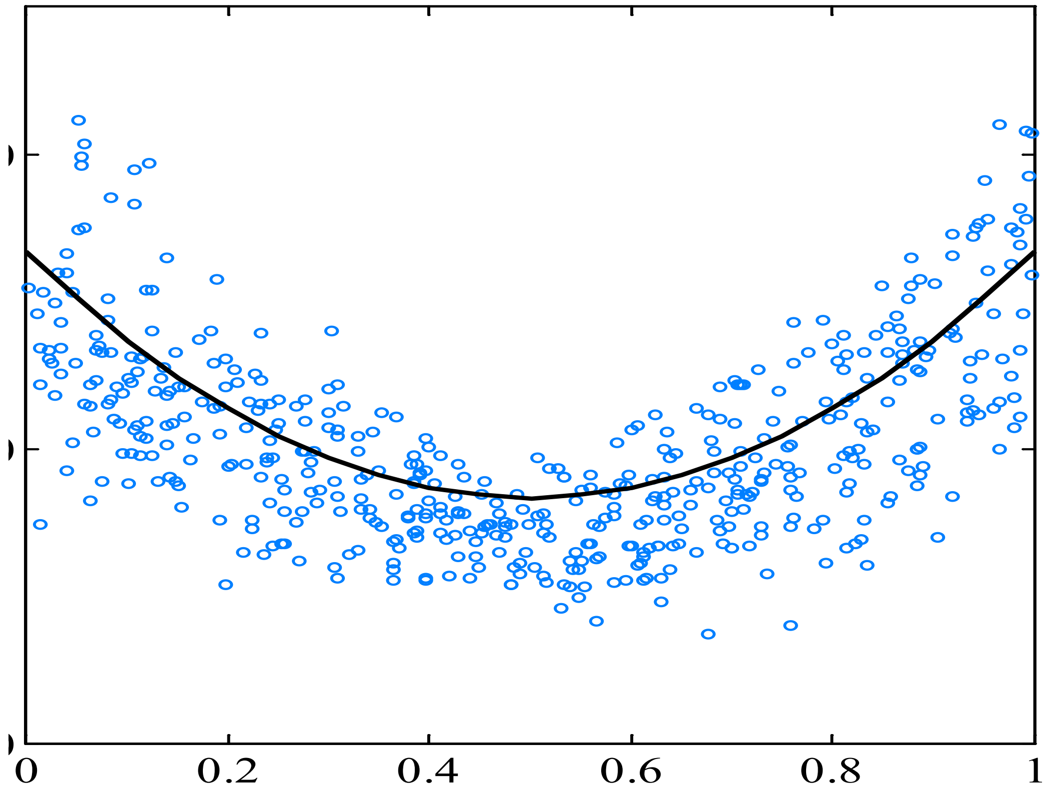
$$\frac{d\theta_i}{dt} = \omega_i + \frac{\sigma}{\langle k \rangle} \sum_{i=1}^{N} a_{ij} \sin(\theta_j - \theta_i),$$

$$\Omega_{ij}^{\alpha} = a_{ij} |\omega_i - \omega_j|^{\alpha}$$



Making your network to explode II : weighting method

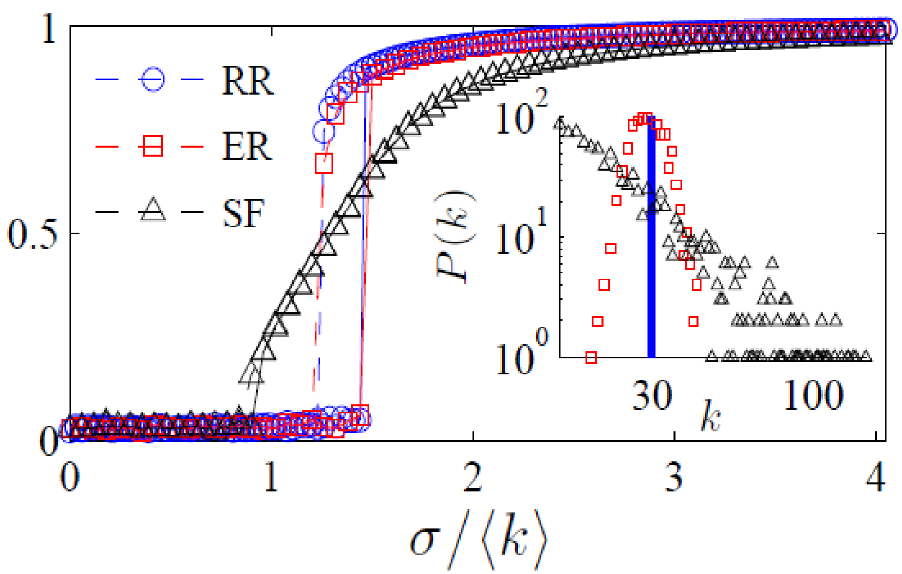
Node strength $$S_i = \sum_j \Omega_{ij}$$



Si

Wi

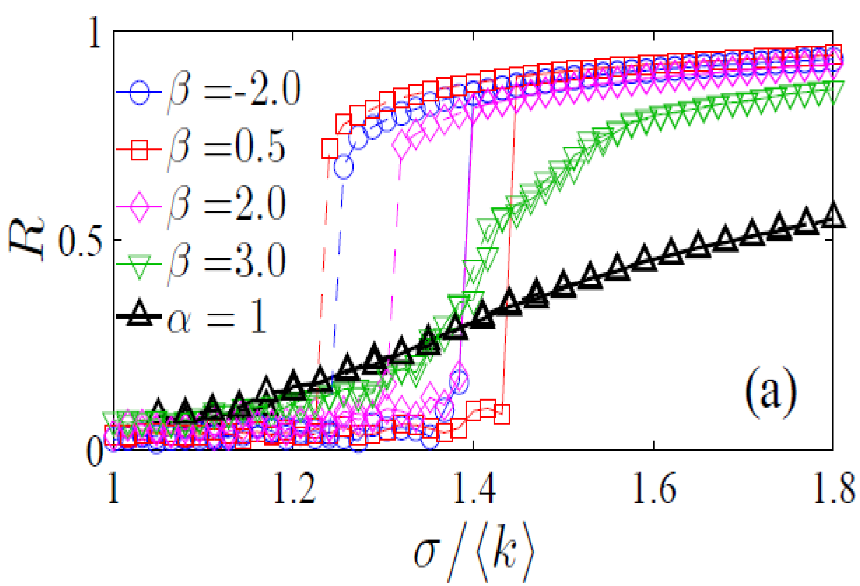
ES and the heterogeneity paradox



Heterogeneous networks need a ***detunning/topology*** weighting:

$$\widetilde{\Omega}_{ij} = a_{ij}|\omega_i - \omega_j| \frac{\ell_{ij}^{\beta}}{\sum_{j \in \mathcal{N}_i} \ell_{ij}^{\beta}}$$

ℓ_{ij} edge betweeness of a_{ij}



Explosive syncronization for □□0

(maximum hysteresis width □=0.5)

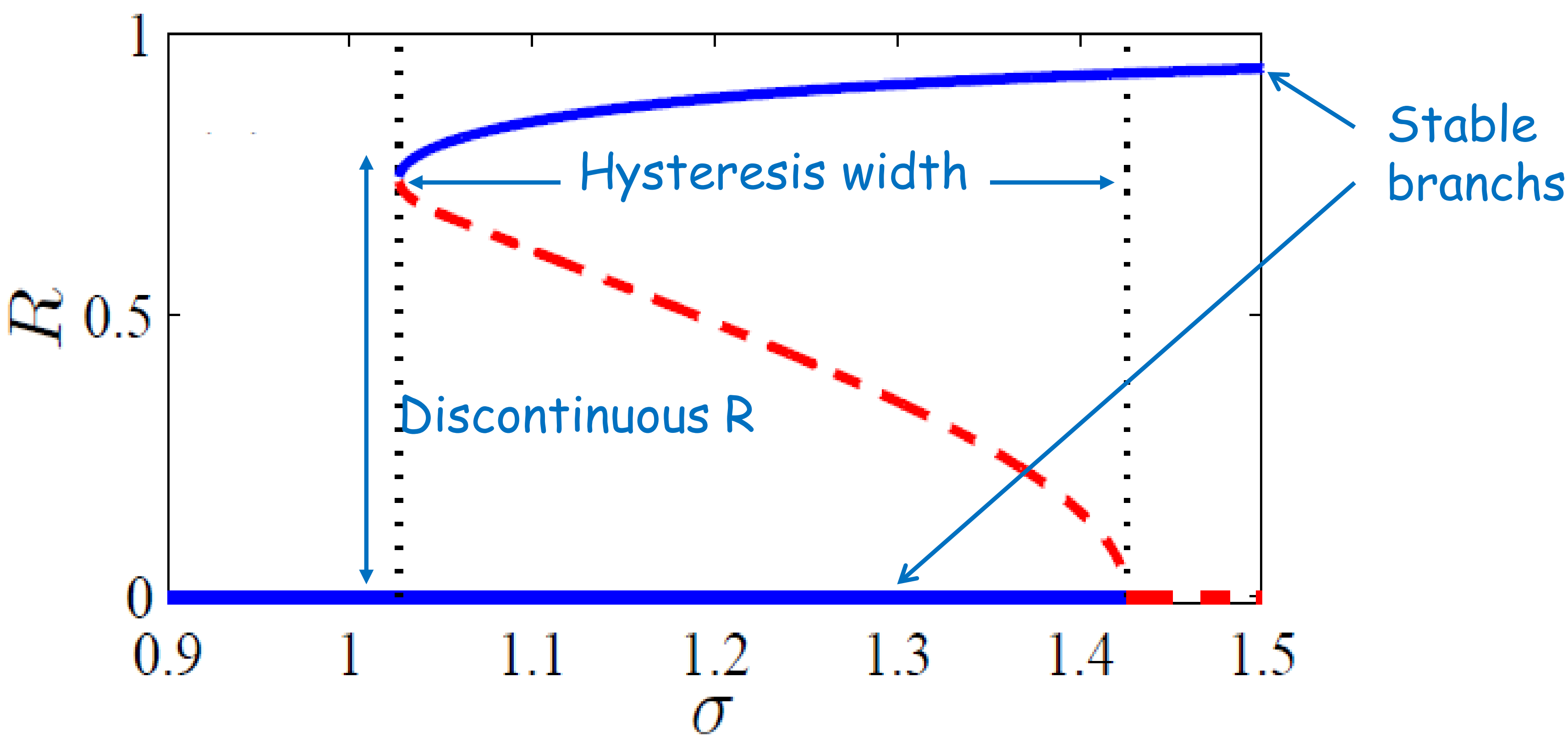
PHYSICAL REVIEW E 88, 042808 (2013)

Analytical results

In the thermodynamic limit $\dot{\theta}_i = \omega_i + \frac{\sigma}{N}\sum_{j=1}^{N}\Omega_{ij}\sin(\theta_j - \theta_i),$

Co-rotating frame phases $\omega = \sigma A_\omega \sin(\theta_\omega - \phi_\omega).$

where $A_\omega \sin\phi_\omega = \int g(x)|w - x|\sin\theta(x)\,dx$ depends on ?



Are correlations necessary? Answer is NO!

CSG

Explosive Synchronization in adaptive and multi-layer networks

X. Zhang, S. Boccaletti, S. Guan, Z. Liu, Phys. Rev. Lett. **114**, 038701 (2015)

$$\Theta'_i = \omega_i + \lambda\alpha_i \sum_{j=1}^{N} A_{ij} \sin(\theta_j - \theta_i)$$

See.....pdf....

What after? The Bellerophon states

Coexistence of quantized, time dependent clusters in globally coupled oscillators

H. Bi, X. Hu, S. Boccaletti, X. Wang, Y. Zou, Z. Liu and S Guan,
Phys. Rev. Lett. **117**, 204101 (2016)

Reading about

Synchronization

* L. Pecora, T. Carroll. Phys. Rev. Lett. **64**, 821 (1990)
* M. Rosenblum, A. Pikovsky, J. Kurths. Phys. Rev. Lett. **76**, 1804–1807 (1996)
*S Boccaletti, J Kurths, G Osipov, DL Valladares, CS Zhou. Phys. Reps **366**, 1 (2002)

Complex networks

*S Boccaletti, V Latora, Y Moreno, M Chavez, DU Hwang. "Coplex networks: structure and dynamics", Phys. Rep. **424**, 175 (2006)
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Explosive synchronization in Complex Networks

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*I. Leyva, et. al. Phys. Rev E. **88**, 042808 (2013)
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